# Math 1B Section 107 Quiz #3

Thursday, 13 September 2007

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Name: ______________________________

1. (3 pts) Say we want to find

$$\int_1^3 \frac{\cos x}{x}\,dx$$

to within an error of 0.001 using the trapezoid rule. What's a reasonable number of intervals into which to divide the domain $[1,3]$? Your number should be big enough to guarantee that the value of the approximation is within the allowed error, but fewer intervals means less computation for the computer.

$$\begin{aligned}
\text{Error(trapezoid)} &\leq \tfrac{K(b-a)^3}{12n^2} && .5\text{ pt}\\
\text{We need } K &\geq |f''(x)| \\
\left|\tfrac{d^2}{dx^2}\left[\tfrac{\cos x}{x}\right]\right| &= \left|\tfrac{d}{dx}\left[-\tfrac{\sin x}{x} - \tfrac{\cos x}{x^2}\right]\right| \\
&= \left|-\tfrac{\cos x}{x} + \tfrac{\sin x}{x^2} + \tfrac{\sin x}{x^2} + \tfrac{2\cos x}{x^3}\right| \\
&\leq \left|\tfrac{\cos x}{x}\right| + \left|\tfrac{2\sin x}{x^2}\right| + \left|\tfrac{2\cos x}{x^3}\right| \\
&\leq \tfrac{1}{x} + \tfrac{2}{x^2} + \tfrac{2}{x^3} \\
&\leq \tfrac{1}{1} + \tfrac{2}{1^2} + \tfrac{2}{1^3} = 5 && 1\text{ pt}\\
\text{So take } K &= 5 \\
\text{Error(trapezoid)} &\leq \tfrac{5(2)^3}{12n^2} \\
\text{We want } n \text{ such that Error(trapezoid)} &\leq 1/1000 \\
\text{This certainly happens if } n &\geq \sqrt{\tfrac{5(2)^3}{12/1000}} && 1\text{ pt}\\
\text{For instance, } n &= \boxed{70} \text{ works.} && .5\text{ pt}
\end{aligned}$$

2. (3 pts) Evaluate the integral

$$\int_{-1}^{1} e^{e^{-x}-x} dx$$

$$u = e^{-x} \quad du = -e^{-x} dx \qquad \text{1 pt}$$

$$\begin{aligned}
\int_{-1}^{1} e^{e^{-x}-x} dx &= \int_{-1}^{1} e^{e^{-x}} e^{-x} dx \\
&= \int_{u=e}^{1/e} -e^{u} du && \text{1 pt} \\
&= -e^{u}\Big|_{u=e}^{1/e} \\
&= \boxed{e^{e} - e^{1/e}} && \text{1 pt}
\end{aligned}$$

3. (4 pts) Evaluate the integral

$$\int_{0}^{1} \ln(x^2+1)\, dx$$

$$\begin{array}{rclcrcl}
u &=& \ln(x^2+1) & & dv &=& dx \\
du &=& 2x/(x^2+1) & & v &=& x
\end{array} \qquad \text{1 pt}$$

$$\begin{aligned}
\int_{0}^{1} \ln(x^2+1)\, dx &= \left[x\ln(x^2+1)\right]_{x=0}^{1} - \int_{0}^{1} \frac{2x^2\, dx}{x^2+1} \\
&= \ln 2 - \int_{0}^{1} \left[2 - \frac{2}{x^2+1}\right] dx && \text{1 pt} \\
&= \ln 2 - 2 + 2\int_{0}^{1} \frac{dx}{x^2+1} \\
&= \ln 2 - 2 + [\arctan(x)]_{x=0}^{1} && \text{1 pt} \\
&= \ln 2 - 2 + \arctan(1) \\
&= \boxed{\ln 2 - 2 + \pi/4} && \text{1 pt}
\end{aligned}$$