Math 1B Section 112 Quiz #3

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Name: ____

1. (3 pts) Say we want to find

$$\int_1^3 \frac{\sin x}{x}\, dx$$

to within an error of 0.001 using the trapezoid rule. What's a reasonable number of intervals into which to divide the domain $[1,3]$? Your number should be big enough to guarantee that the value of the approximation is within the allowed error, but fewer intervals means less computation for the computer.

$$\begin{aligned}
\text{Error(trapezoid)} &\leq \tfrac{K(b-a)^3}{12n^2} && .5 \text{ pt} \\
\text{We need } K &\geq |f''(x)| \\
\left|\tfrac{d^2}{dx^2}\left[\tfrac{\sin x}{x}\right]\right| &= \left|\tfrac{d}{dx}\left[\tfrac{\cos x}{x} - \tfrac{\sin x}{x^2}\right]\right| \\
&= \left|-\tfrac{\sin x}{x} - \tfrac{\cos x}{x^2} - \tfrac{\cos x}{x^2} + \tfrac{2\sin x}{x^3}\right| \\
&\leq \left|\tfrac{\sin x}{x}\right| + \left|\tfrac{2\cos x}{x^2}\right| + \left|\tfrac{2\sin x}{x^3}\right| \\
&\leq \tfrac{1}{x} + \tfrac{2}{x^2} + \tfrac{2}{x^3} \\
&\leq \tfrac{1}{1} + \tfrac{2}{1^2} + \tfrac{2}{1^3} = 5 && 1 \text{ pt} \\
\text{So take } K &= 5 \\
\text{Error(trapezoid)} &\leq \tfrac{5(2)^3}{12n^2} \\
\text{We want } n \text{ such that Error(trapezoid)} &\leq 1/1000 \\
\text{This certainly happens if } n &\geq \sqrt{\tfrac{5(2)^3}{12/1000}} && 1 \text{ pt} \\
&\leq \boxed{70} \text{ for example.} && .5 \text{ pt}
\end{aligned}$$

2. (3 pts) Evaluate the integral

$$\int_0^1 e^{e^{-x}-x} dx$$

$$u = e^{-x} \quad du = -e^{-x}dx \qquad \text{1 pt}$$

$$\begin{aligned} \int_0^1 e^{e^{-x}-x} dx &= \int_0^1 e^{e^{-x}} e^{-x} dx \\ &= \int_{u=1}^{1/e} -e^u du && \text{1 pt} \\ &= -e^u \big|_{u=1}^{1/e} \\ &= \boxed{e - e^{1/e}} && \text{1 pt} \end{aligned}$$

3. (4 pts) Evaluate the integral

$$\int_{-1}^1 \ln(x^2+1)\, dx$$

$$\begin{aligned} u &= \ln(x^2+1) & dv &= dx \\ du &= 2x/(x^2+1) & v &= x \end{aligned} \qquad \text{1 pt}$$

$$\begin{aligned} \int_{-1}^1 \ln(x^2+1)\, dx &= \left[x \ln(x^2+1)\right]_{x=-1}^1 - \int_{-1}^1 \frac{2x^2\, dx}{x^2+1} \\ &= 2\ln 2 - \int_{-1}^1 \left[2 - \frac{2}{x^2+1}\right] dx && \text{1 pt} \\ &= 2\ln 2 - 4 + 2\int_{-1}^1 \frac{dx}{x^2+1} \\ &= 2\ln 2 - 4 + [\arctan(x)]_{x=-1}^1 && \text{1 pt} \\ &= 2\ln 2 - 4 + \arctan(1) - \arctan(-1) \\ &= \boxed{2\ln 2 - 4 + \pi/2} && \text{1 pt} \end{aligned}$$