# Math 1B Section 107 Quiz #4

Thursday, 20 September 2007

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Name: ______________________________________

1. (3 pts) Let's say I believe that $y$ is a function of $x$, and I do an experiment and get the following values of $x$ and $y$:

| $x$ | $y(x)$ | $y''(x)$ |
|---|---|---|
| 0 | 0 | — |
| 1 | 0 | 1 |
| 2 | 1 | 0 |
| 3 | 2 | 1 |
| 4 | 4 | — |

For the particular measurement I'm making, I want to estimate $\int_0^4 y(x)dx$. If I use the trapezoid approximation with $n = 4$, I get

$$\int_0^4 y(x)dx \approx 5.$$

How good of an estimate is this? I.e. what is the expected error? You can't evaluate the second derivative $y''(x)$ exactly, but you can estimate it: if $y(x-1) = a$, $y(x) = b$, and $y(x+1) = c$, then $y''(x) \approx a - 2b + c$. So estimate $y''(1)$, $y''(2)$, and $y''(3)$, and use these to estimate the error $E_4^T$.

$E_n^T \lesssim \frac{K(b-a)^3}{12n^2}$ 1 pt

$K \gtrsim \max|y''(x)| = 1$ 1 pt

$E_4^T \lesssim \frac{1\times 4^3}{12\times 4^2} = \boxed{1/3}$ 1 pt

Determine whether the following definite integrals are convergent or divergent. Evaluate each convergent integral.

2. (3 pts) $\displaystyle\int_0^2 \frac{dx}{x\sqrt{x}}$

$$= \int_0^2 \frac{dx}{x^{3/2}}$$

$p = 3/2 \geq 1$ 1 pt

So integral $\boxed{\text{diverges}}$ by the $p$-Test. 2 pt

3. (4 pts) $\displaystyle\int_1^\infty \frac{x-2}{x^3+3x^2+2x}\,dx$

$$\frac{x-2}{x^3+3x^2+2x} \leq \frac{x}{x^3} = \frac{1}{x^2},$$ which converges by $p$-Test.

So integral converges by Comparison Test. 2 pt

$$\frac{x-2}{x^3+3x^2+2x} = \frac{-1}{x} + \frac{3}{x+1} + \frac{-2}{x+2}$$ 1 pt

$$\int_1^\infty \frac{(x-2)\,dx}{x^3+3x^2+2x} = \lim_{t\to\infty} \int_1^t \frac{(x-2)\,dx}{x^3+3x^2+2x}$$

$$= \lim_{t\to\infty} \left[-\ln(x) + 3\ln(x+1) - 2\ln(x+2)\right]_1^t$$ .5 pt

$$= \lim_{t\to\infty} \left[\ln\left(\frac{(x+1)^3}{x(x+2)^2}\right)\right]_1^t$$

$$= \lim_{t\to\infty} \left[\ln\left(\frac{(t+1)^3}{t(t+2)^2}\right) - \ln\left(\frac{(1+1)^3}{1(1+2)^2}\right)\right]$$

$$= \ln(1) - \ln\left(\frac{2^3}{3^2}\right) = \boxed{\ln(9/8)}$$ .5 pt