

Math 1B Section 112 Quiz #4

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Name: _____

1. (3 pts) Let's say I believe that y is a function of x , and I do an experiment and get the following values of x and y :

x	$y(x)$	$y''(x)$
0	0	—
1	0	1
2	1	1
3	3	-1
4	4	—

For the particular measurement I'm making, I want to estimate $\int_0^4 y(x)dx$. If I use the trapezoid approximation with $n = 4$, I get

$$\int_0^4 y(x)dx \approx 6.$$

How good of an estimate is this? I.e. what is the expected error? You can't evaluate the second derivative $y''(x)$ exactly, but you can estimate it: if $y(x-1) = a$, $y(x) = b$, and $y(x+1) = c$, then $y''(x) \approx a - 2b + c$. So estimate $y''(1)$, $y''(2)$, and $y''(3)$, and use these to estimate the error E_4^T .

$$E_n^T \lesssim \frac{K(b-a)^3}{12n^2} \qquad 1 \text{ pt}$$

$$K \gtrsim \max |y''(x)| = 1 \qquad 1 \text{ pt}$$

$$E_4^T \lesssim \frac{1 \times 4^3}{12 \times 4^2} = \boxed{1/3} \qquad 1 \text{ pt}$$

Determine whether the following definite integrals are convergent or divergent. Evaluate each convergent integral.

2. (3 pts) $\int_0^1 \frac{dx}{x\sqrt{x}}$

$$= \int_0^1 \frac{dx}{x^{3/2}}$$

$$p = 3/2 \geq 1$$

1 pt

So integral diverges by the p -Test.

2 pt

3. (4 pts) $\int_1^\infty \frac{x-4}{x^3+3x^2+2x} dx$

$$\frac{x-4}{x^3+3x^2+2x} \leq \frac{x}{x^3} = \frac{1}{x^2}, \text{ which converges by } p\text{-Test.}$$

So integral converges by Comparison Test.

2 pt

$$\frac{x-4}{x^3+3x^2+2x} = \frac{-2}{x} + \frac{5}{x+1} + \frac{-3}{x+2}$$

1 pt

$$\int_1^\infty \frac{(x-4) dx}{x^3+3x^2+2x} = \lim_{t \rightarrow \infty} \int_1^t \frac{(x-4) dx}{x^3+3x^2+2x}$$

$$= \lim_{t \rightarrow \infty} [-2 \ln(x) + 5 \ln(x+1) - 3 \ln(x+2)]_1^t$$

.5 pt

$$= \lim_{t \rightarrow \infty} \left[\ln \left(\frac{(x+1)^5}{x^2(x+2)^3} \right) \right]_1^t$$

$$= \lim_{t \rightarrow \infty} \left[\ln \left(\frac{(t+1)^5}{t^2(t+2)^3} \right) - \ln \left(\frac{(1+1)^5}{1^2(1+2)^3} \right) \right]$$

$$= \ln(1) - \ln \left(\frac{2^5}{3^3} \right) = \boxed{\ln(27/32)}$$

.5 pt