Math 1B Section 112 Quiz #5

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Name: _____

1. (3 pts) Find the length of the curve $y = \ln(\cos x)$ as x ranges between 0 and $\pi/4$.

$$\begin{aligned}
\text{length of curve} &= \int_0^{\pi/4} \sqrt{1 + (y')^2}\, dx && \text{.5 pt} \\
&= \int_0^{\pi/4} \sqrt{1 + (-\sin x/\cos x)^2}\, dx && \text{.5 pt} \\
&= \int_0^{\pi/4} \sec x\, dx && \text{1 pt} \\
&= \ln(\sec x + \tan x)\big|_0^{\pi/4} && \text{.5 pt} \\
&= \ln(2/\sqrt{2} + 1) - \ln(1) && \text{.5 pt} \\
&= \ln(1 + \sqrt{2})
\end{aligned}$$

2. (3 pts) Find the area of the surface of revolution generated by rotating the curve $y = x^3$, as x ranges between 1 and 3, around the x-axis.

$$\begin{aligned}
\text{area of surface} &= 2\pi \int_1^3 y\sqrt{1 + (y')^2}\, dx && \text{.5 pt} \\
&= 2\pi \int_{x=1}^3 x^3\sqrt{1 + 9x^4}\, dx && \text{.5 pt} \\
&= 2\pi \int_{u=10}^{1+9\cdot 3^4} \sqrt{u}\, \tfrac{du}{36} && \text{1 pt} \\
&= \tfrac{\pi}{18}\tfrac{2}{3}\, u^{3/2}\big|_{10}^{270} && \text{1 pt} \\
&= \tfrac{\pi}{27}(730^{3/2} - 10^{3/2})
\end{aligned}$$

3. (4 pts) Find the area of the surface of revolution generated by rotating the curve $y = \dfrac{e^x + e^{-x}}{2} = \cosh x$, as x ranges between 0 and 2, around the y-axis.

$$\begin{aligned}
\text{area of surface} &= 2\pi \textstyle\int_0^2 x\sqrt{1+(y')^2}\,dx && \text{.5 pt}\\
&= 2\pi \textstyle\int_0^2 x\sqrt{1+\left(\frac{e^x-e^{-x}}{2}\right)^2}\,dx && \text{.5 pt}\\
&= 2\pi \textstyle\int_0^2 x\sqrt{\frac{2+e^{2x}-1+e^{-2x}}{4}}\,dx &&\\
&= 2\pi \textstyle\int_0^2 x\left(\frac{e^x+e^{-x}}{2}\right)dx && \text{1 pt}\\
&= \pi \textstyle\int_0^2 xe^x\,dx + \pi\int_0^2 xe^{-x}dx &&\\
&= \pi\left[xe^x - e^x\right]_0^2 + \pi\left[-xe^{-x} - e^{-x}\right]_0^2 && \text{1.5 pt}\\
&= \pi\left(e^2 + 1 - 3e^{-2} + 1\right) = 2\pi + \pi e^2 - 3\pi/e^2 && \text{.5 pt}
\end{aligned}$$