Math 1B Section 107 Quiz #6

Thursday, 4 October 2007

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Name: ______________________________________

1. **True or False** (1 pt each) For each of the following statements, decide if it is true or false. You do not need to show work: I will grade only your answers.

   (a) If a sequence $\{a_n\}_{n=1}^{\infty}$ is strictly *increasing*, and there's a number M bounding the sequence from *below* (i.e. $a_n \geq M$ for every n), then $\lim_{n\to\infty} a_n$ exists.

   False *A sequence bounded below can nontheless grow to infinity: indeed, any increasing sequence is bounded below.*

   (b) If a sequence $\{a_1, a_2, a_3, \ldots\}$ converges, then the sequence $\{b_n\} = \{a_{2n}\} = \{a_2, a_4, a_6, \ldots\}$ converges to the same limit.

   True *If the entire sequence gets close to number, then certainly the even terms get close to that number.*

   (c) A geometric series cannot converge if the ratio between successive terms is negative.

   False *A geometric series converges if the ratio, negative or positive, has absolute value less than* 1.

2. (3 pts) Use the divergence test to show that the following series diverges. (You will need to actually compute a limit, or explain why the limit is not defined.)

$$\sum_{n=1}^{\infty} \frac{n^2}{3n^2+1}$$

We use the divergence test: the limit $\lim_{n\to\infty}\left(\frac{n^2}{3n^2+1}\right) = \lim_{n\to\infty}\left(\frac{1}{3} - \frac{1/3}{3n^2+1}\right) = \frac{1}{3} - \lim_{n\to\infty}\left(\frac{1/3}{3n^2+1}\right) = \frac{1}{3} - 0 \neq 0$, *so the series cannot converge.*

3. (4 pts) Sum the following telescoping series:

$$\sum_{n=1}^{\infty} \frac{2}{(2n-1)(2n+1)} = \frac{2}{3} + \frac{2}{15} + \frac{2}{35} + \dots$$

$$\begin{aligned} \frac{2}{(2n-1)(2n+1)} &= \frac{1}{2n-2} - \frac{1}{2n+1} \\ \sum_{n=1}^{\infty} \frac{2}{(2n-1)(2n+1)} &= \frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \dots \\ &= 1 \end{aligned}$$