Math 1B Section 112 Quiz #6

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Name: ______________________________________

1. **True or False** (1 pt each) For each of the following statements, decide if it is true or false. You do not need to show work: I will grade only your answers.

   (a) If a sequence $\{a_n\}_{n=1}^{\infty}$ is strictly *increasing*, and there's a number M bounding the sequence from *above* (i.e. $a_n \leq M$ for every n), then $\lim_{n\to\infty} a_n$ exists.

   True *This is the (an equivalent) statement of the monotonic sequences theorem.*

   (b) Let $f(x)$ be a function, and define the sequence $a_n = f(n)$. If $\lim_{n\to\infty} a_n = L$, then $\lim_{x\to\infty} f(x) = L$.

   False *It's true that if* $\lim_{x\to\infty} f(x)$ *exists, then this limit equals* $\lim_{n\to\infty} a_n$ *(which necessarily converges). But just because a sequence converges does not mean that the function converges (e.g.* $f(x) = \sin(\pi x)$*).*

   (c) A geometric series converges if and only if the ratio between successive terms is positive.

   False *The ratio must be strictly more than* -1 *and strictly less than* $+1$*.*

2. (3 pts) Use the divergence test to show that the following series diverges. (You will need to actually compute a limit, or explain why the limit is not defined.)

$$\sum_{n=1}^{\infty} \frac{n^3}{2n^3+1}$$

We use the divergence test: the limit $\lim_{n\to\infty}\left(\frac{n^3}{2n^3+1}\right) = \lim_{n\to\infty}\left(\frac{1}{2} + \frac{-1/4}{2n^3+1}\right) = \frac{1}{2} + \lim_{n\to\infty}\left(\frac{-1/2}{2n^3+1}\right) = \frac{1}{2} + 0 \neq 0$. *Thus, since the limit is not* 0, *the series necessarily diverges.*

3. (4 pts) Sum the following telescoping series:

$$\sum_{n=1}^{\infty} \frac{3}{(3n-2)(3n+1)} = \frac{3}{4} + \frac{3}{28} + \frac{3}{70} + \ldots$$

$$\begin{aligned}
\frac{3}{(3n-2)(3n+1)} &= \frac{1}{3n-2} - \frac{1}{3n+1} \\
\sum_{n=1}^{\infty} \frac{3}{(3n-2)(3n+1)} &= \frac{1}{1} - \frac{1}{4} + \frac{1}{4} - \frac{1}{7} + \frac{1}{7} - \frac{1}{10} + \ldots \\
&= 1
\end{aligned}$$