Math 1B Section 107 Quiz #8

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For the first two questions, the alternating series diverge. For each series, decide which parts of the Alternating Series Test it satisfies, and which parts it fails to satisfy:

1. (2 pts) $\sum_{n=1}^{\infty}(-1)^n \frac{n+1}{2n}$

   The alternating series test demands that a series be

   (a) *alternating: either* $\sum(-1)^n a_n$ *or* $\sum(-1)^{n-1} a_n$, *where* $a_n \geq 0$.

   (b) *decreasing:* $a_n \geq a_{n+1}$

   (c) *tending towards 0:* $\lim_{n\to\infty} a_n = 0$.

   This series satisfied (a) and (b), but fails to have the correct limit:

$$\lim_{n\to\infty} \frac{n+1}{2n} = \frac{1}{2}$$

2. (2 pts) $1 - \frac{1}{4} + \frac{1}{3} - \frac{1}{16} + \frac{1}{5} - \frac{1}{64} + \frac{1}{7} - \frac{1}{256} + \frac{1}{9} - \frac{1}{1024} + \frac{1}{11} - \frac{1}{4096} + \dots$

   This series satisfies parts (a) and (c) above, but fails to satisfy part (b); for example, $\frac{1}{5} > \frac{1}{16}$, *and* $\frac{1}{7} > \frac{1}{64}$.

For the next two questions, use the **Ratio Test** to determine if the series converges or diverges.

3. (3 pts) $\sum_{n=0}^{\infty} n! \left(\frac{1}{4}\right)^n$

   We use the ratio test:

$$\lim_{n\to\infty} \left| \frac{(n+1)! \left(\frac{1}{4}\right)^{n+1}}{n! \left(\frac{1}{4}\right)^n} \right| = \lim_{n\to\infty} \left| (n+1)\frac{1}{4} \right| = +\infty > 1$$

   so the series diverges.

4. (3 pts) $\sum_{n=0}^{\infty} \frac{n}{2^n + 1}$

   We use the ratio test:

$$\lim_{n\to\infty} \left| \frac{\frac{n+1}{2^{n+1}+1}}{\frac{n}{2^n+1}} \right| = \lim_{n\to\infty} \left| \frac{n+1}{n} \frac{2^n+1}{2^{n+1}+1} \right| = \lim_{n\to\infty} \left| \frac{1+\frac{1}{2^n}}{2+\frac{1}{2^n}} \right| = \frac{1}{2} < 1$$

   so the series converges absolutely.