Math 1B Section 112 Quiz #8

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For the first two questions, the alternating series diverge. For each series, decide which parts of the Alternating Series Test it satisfies, and which parts it fails to satisfy:

1. (2 pts) $\sum_{n=1}^{\infty}(-1)^n \frac{n+1}{2n}$

   The alternating series test demands that a series be

   (a) alternating: either $\sum(-1)^n a_n$ *or* $\sum(-1)^{n-1}a_n$*, where* $a_n \geq 0$*.*

   (b) decreasing: $a_n \geq a_{n+1}$

   (c) tending towards 0: $\lim_{n\to\infty} a_n = 0$*.*

   This series satisfied (a) and (b), but fails to have the correct limit:

   $$\lim_{n\to\infty} \frac{n+1}{2n} = \frac{1}{2}$$

2. (2 pts) $1 - \frac{1}{4} + \frac{1}{3} - \frac{1}{16} + \frac{1}{5} - \frac{1}{64} + \frac{1}{7} - \frac{1}{256} + \frac{1}{9} - \frac{1}{1024} + \frac{1}{11} - \frac{1}{4096} + \dots$

   This series satisfies parts (a) and (c) above, but fails to satisfy part (b); for example, $\frac{1}{5} > \frac{1}{16}$*, and* $\frac{1}{7} > \frac{1}{64}$*.*

For the next two questions, use the **Ratio Test** to determine if the series converges or diverges.

3. (3 pts) $\displaystyle\sum_{n=0}^{\infty} \frac{n+1}{3^n}$

   We use the ratio test:

$$\lim_{n\to\infty} \left| \frac{\frac{n+2}{3^{n+1}}}{\frac{n+1}{3^n}} \right| = \lim_{n\to\infty} \left| \frac{n+2}{n+1} \frac{1}{3} \right| = \frac{1}{3} < 1$$

   so the series converges absolutely.

4. (3 pts) $\displaystyle\sum_{n=0}^{\infty} n! \left(\frac{1}{3}\right)^n$

   We use the ratio test:

$$\lim_{n\to\infty} \left| \frac{(n+1)! \left(\frac{1}{3}\right)^{n+1}}{n! \left(\frac{1}{3}\right)^n} \right| = \lim_{n\to\infty} \left| (n+1) \frac{1}{3} \right| = +\infty > 1$$

   so the series diverges.