

Math 1B Quiz #10

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Name: _____

1. (3 pts) Show that $f(t) = Ce^t - t - 1$ is a one-parameter family of solutions to the differential equation

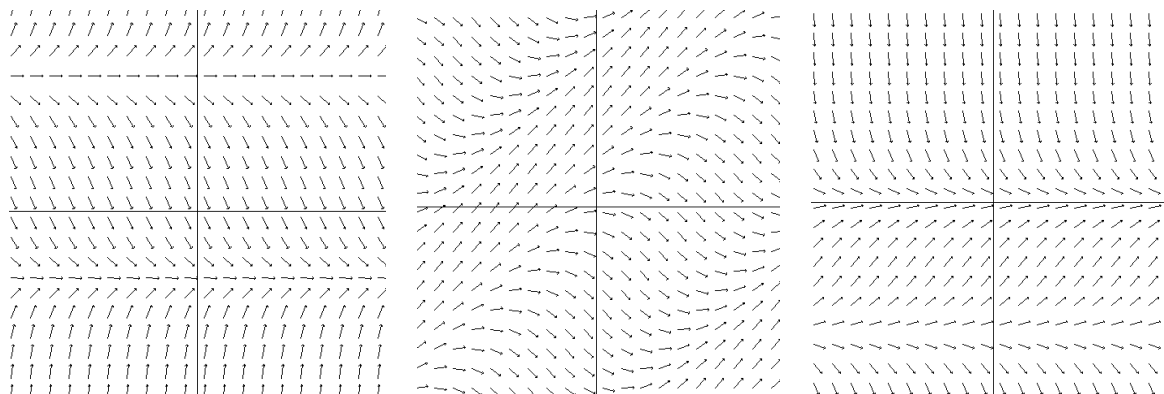
$$\frac{df}{dt} = f(t) + t.$$

If $f(0) = 1$, what is $f(t)$?

2. (3 pts) Which of the three direction fields corresponds to the differential equation

$$\frac{df}{dt} = (f + 1)(f - 2)?$$

Each is graphed with f and t ranging from -3 to 3 . Based on the graph (or any other way), specify which values of f are equilibrium values. Which of these equilibria are stable and which are unstable as $t \rightarrow \infty$? Sketch the solution to this differential equation that satisfies the initial value condition that $f = 0$ when $t = 0$.



3. (4 pts) Set up, but *do not solve*, a differential equation modeling the following rather unlikely situation. Be sure to specify the meanings of your symbols: you should provide a “dictionary” that translates the interesting quantities into your chosen variables (“ $h = \text{height}$,” for instance). Also be sure to specify the initial condition for the problem.

Dr. Gregory House’s evil twin Dr. Geoffrey Condo breaks into the set of *Grey’s Anatomy*, hoping to murder his brother’s arch rival Dr. Meredith Grey. He successfully restrains her to a hospital bed, and implants her with an intravenous drug drip, which he programs to administer an increasing supply of painkillers: 100 miligrams this hour, then 200 mg, and so on, so that the input rate of painkiller is proportional to the amount of time since she was incapacitated. Dr. Condo hopes that by doing this, Dr. Grey will overdose on painkillers.

But Dr. Grey’s kidneys decide to try to keep her from dying, and filter out the painkiller (assuming the IV also administers plenty of water at the right salinity). In particular, in an hour, her kidneys can filter out one tenth of the total painkillers in her body. How much painkiller is in Dr. Grey’s body after a given amount of time?