Math 1B Quiz #11

Thursday, 15 November 2007

GSI: Theo Johnson-Freyd
http://math.berkeley.edu/∼theojf

Name: _____

1. (3 pts) Solve the following initial-value problem, to write f as a function of t:

$$\frac{df}{dt} = t \cdot \left(f^2 + 1\right), \quad f(2) = 0$$

2. (3 pts) Solve the following differential equation, to find a one-parameter family of solutions for y as a function of x:

$$2xy' + y = 6x$$

3. (4 pts) Carbon has two stable isotopes — carbon-12 (^{12}C) and carbon-13 (^{13}C) — and one relatively common radioactive isotope carbon-14 (^{14}C), produced in the upper atmosphere by bombardment with cosmic radiation. Plants absorb atmospheric carbon, and hence the concentration of ^{14}C in plants is equal to the atmospheric concentration. When plants die, they do not absorb any new carbon. The amount of ^{14}C in archeological samples is used to date archeological sites.

   (a) Like all radioactive materials, ^{14}C decays at a constant relative rate: the amount that decays in any given period of time is proportional to the amount present. Write a differential equation modeling the amount of ^{14}C in a given amount of time.

   (b) The half-life of ^{14}C is 5730 years, and the atmospheric concentration of ^{14}C is 600 billion atoms per mole (roughly one part per trillion). What is the solution to your differential equation (relating how much time has elapsed with the amount of ^{14}C left)? You do not need to simplify, but you do need to use units.

   (c) A sample from Fell's Cave, in southern Chile, has a ^{14}C concentration of 150 billion atoms per mole. Roughly what is the date of the archeological site?

   (d) What is the rate of radioactive decay of $^{14}\text{C} \to {}^{12}\text{C}$ in the sample if the current concentration of ^{14}C is 150 billion atoms per mole? You do not need to simplify, but you do need to report units.