Math 1B Worksheet 1:
Review of 1A

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Please introduce yourselves to each other, and put your names at the top of a piece of blackboard. Take turns being the scribe: each of you should have a chance to write on the chalkboard for at least one of the exercises.

These exercises are hard — harder than on the homework, quizzes, or exams. That means that you should spend some time thinking and talking about them; they're designed to be solved in groups (the best way to learn mathematics). The problems are roughly in order of increasing difficulty. I don't expect any group to solve all of them.

Here's a hint: drawing pictures — sketching graphs of functions — will always make the problem easier.

1. Writing $|x|$ for the absolute value of x $(= x$ is $x \geq 0$ and $= -x$ if $x \leq 0)$, at what points is the function

$$f(x) = |-1 + |-1 + |-1 + |-1 + |-1 + |x||||||$$

   continuous? Differentiable? Where is it differentiable, compute its derivative.

2. Compute $\displaystyle\lim_{x \to \infty} \sqrt{\frac{9x^3 - 4x}{4x^3 + x^2 + 1}} - \frac{100}{x}$.

3. What is the slope of the function $f(x) = x^2 \sin x$ at $x = 0$? At $x = \pi/2$? At how many places does $f(x)$ achieve a local maximum or minimum?

4. Compute $\dfrac{d}{dx} \left[\ln \left(\sin(x^2 + 1) + 2 \right) \right]$.

5. Compute $\displaystyle\int_{x=\sqrt{2\pi-1}}^{\sqrt{(5\pi/2)-1}} \left(e^{\cos(x^2+1)} \sin(x^2 + 1) - e^{\sin(x^2+1)} \cos(x^2 + 1) \right) x \, dx$.

6. What is $\displaystyle\int e^x \cos x \, dx$? How about $\displaystyle\int e^{ax} \cos x \, dx$, where a is a constant? How about $\displaystyle\int x \, e^x \cos x \, dx$?

7. What's wrong with the following proof that $0 = 1$?

$$\ln x = \int \frac{1}{x} \, dx = \frac{1}{x} x - \int \frac{-1}{x^2} x \, dx = 1 + \int \frac{1}{x} \, dx = 1 + \ln x$$