Math 1B Worksheet 17:
More Taylor Series

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Please introduce yourselves to each other, and put your names at the top of a piece of blackboard. Take turns being the scribe: each of you should have a chance to write on the chalkboard for at least one of the exercises.

These exercises are hard — harder than on the homework, quizzes, or exams. That means that you should spend some time thinking and talking about them; they're designed to be solved in groups (the best way to learn mathematics). The problems are roughly in order of increasing difficulty. I don't expect any group to solve all of them.

Don't forget to draw pictures.

1. (a) Let's say you have a series $a(x) = \sum a_n (x-7)^n$ with positive radius of convergence. What can you say about the interval of convergence?
   (b) What if you also know that $a(x)$ converges for $x = 3$? For instance, do you know whether it converges at $x = 9$?
   (c) What if you also know that $a(x)$ diverges for $x = 2$? For instance, do you know whether it converges at $x = 11$?

2. Find the power series expansions of the following functions:
   (a) $(3+x)^{1/3}$
   (b) $\ln(5+x)$
   (c) $\arctan(6x)$

   What are their radii of convergence?

3. Recall that we can use the Taylor series to estimate functions: if you add up only the first n terms in the Taylor series, then the error is

$$\left| f(x) - \left(f(0) + f'(0)\,x + \frac{f''(0)\,x^2}{2} + \ldots + \frac{f^{(n)}(0)\,x^n}{n!} \right) \right| \leq \frac{M\,x^{n+1}}{(n+1)!}$$

   where M is any number so that $M \geq \left| f^{(n+1)}(z) \right|$ for all z between 0 and x. Thus, what n should I take if I want to use the Taylor series for e^x (centered at $c = 0$) to evaluate $e^{1/2}$?