Math 1B Worksheet 2:
Intgegration by parts and trigonometry

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Please introduce yourselves to each other, and put your names at the top of a piece of blackboard. Take turns being the scribe: each of you should have a chance to write on the chalkboard for at least one of the exercises.

These exercises are hard — harder than on the homework, quizzes, or exams. That means that you should spend some time thinking and talking about them; they're designed to be solved in groups (the best way to learn mathematics). The problems are roughly in order of increasing difficulty. I don't expect any group to solve all of them.

1. Calculate $\displaystyle\int_{x=0}^{3\pi/2} (\sin x + \cos x)^3 \, dx$.

2. Let a be a number such that $0 < a < \pi/2$. Compute the volume obtained by rotating the region bounded by the curves

$$y = \tan x, \ y = 0, \ x = a$$

about the x-axis.

3. (a) Write down a reduction formula for $\displaystyle\int (\ln x)^n \, dx$.

   (b) Evaluate $\displaystyle\int_0^1 (\ln x)^n \, dx$.

4. Calculate $\displaystyle\int_{x=0}^{\pi/2} \cos^n(x) \, dx$ in two different ways:

   (a) Using a reduction formula. What happens to the boundary terms (the uv in $\int u \, dv = uv - \int v \, du$)? You will have to consider two different cases: the formulas are different when n is odd and when n is even.

   (b) When $n = 2k+1$, use a u-substitution. For any given k, you could then expand out and evaluate the integral. For a general k, you can use integration by parts to get a reduction formula.

5. **$+C$ in integration by parts**

   When we integrate, we usually remember to add an arbitrary constant C to our integrals. Why don't we need to do this when integrating by parts? Integrate $\int x^n e^x dx$ completely honestly: let $u = x^n$ and $dv = e^x dx$, but this time let $v = e^x + C$.