

Math 1B Worksheet 21:

Separable Differential Equations

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Please introduce yourselves to each other, and put your names at the top of a piece of blackboard. Take turns being the scribe: each of you should have a chance to write on the chalkboard for at least one of the exercises.

These exercises are hard — harder than on the homework, quizzes, or exams. That means that you should spend some time thinking and talking about them; they're designed to be solved in groups (the best way to learn mathematics). The problems are roughly in order of increasing difficulty. I don't expect any group to solve all of them.

Don't forget to draw pictures.

1. If you didn't try it last time, the requisite turkey problem:

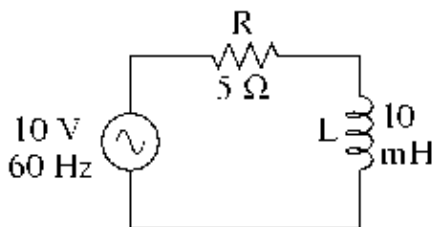
A turkey, which starts at 70 degrees Fahrenheit, is placed in a 300-degree. The rate of heat transfer between two materials is proportional to the difference in temperatures of the materials (the proportionality constant depends on various parameters, like the type of material and the size). The turkey is done when the internal temperature reaches 180 degrees. If after an hour the internal temperature of the turkey is 86 degrees, and dinner is at 5 p.m., when should you start cooking for Thanksgiving?

2. (a) The Pacific halibut fishery, if there is no fishing, can be modeled as follows: the rate of growth is proportional to the total population (with a proportionality constant of 0.71 per year), and also to “the amount of space left” — take the ratio of the population with the total carrying capacity (8×10^7 kg), and subtract from 1. Write and solve a differential equation modeling this situation. Identify the equilibrium states, and specify whether those equilibria are *stable* or *unstable*.
(b) In fact, the US Halibut Fishery is certified by Eat On The Wild Side as meeting the environmental standards set by the Marine Stewardship Council. Let's say you're managing this fishery: your goal is for the fishing companies to harvest exactly as much halibut as is produced in a year (so that the net growth of biomass is zero). You also want to maximize how much halibut you can harvest and sell. What is the optimal population of halibut? How much halibut can you harvest per year? If there are poachers, so that the total harvest stays constant at higher than this amount, what happens to the population of halibut?

- (c) In general, let's say the fishing companies harvest a total of H kg of halibut per year. Model this new situation by solving a differential equation. What are the stable and unstable equilibria?
3. Remember that a series circuit with resistance R , inductance L , and power source $E(t)$ satisfies the linear differential equation

$$L \frac{dI}{dt} + RI = E(t)$$

where $I(t)$ is the current around the circuit. An AC power source provides a sinusoidal voltage: e.g. for this circuit, the power supply provides a voltage $E(t) = (10 \text{ V}) \sin(t \times 60 \text{ Hz})$. Solve the differential equation to determine the current flowing around the loop at a given time.



4. Find the curves perpendicular to the family $x^2 - y^2 = k$. What about $x^2 + y^2 = k$? More generally, if a and b are held constant, and the parameter k is allowed to vary, then $ax^2 + by^2 = k$ is a family of curves; find the family of curves perpendicular to this family.