Math 1B Worksheet 26:
Applications of Second-Order Differential Equations

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Please introduce yourselves to each other, and put your names at the top of a piece of blackboard. Take turns being the scribe: each of you should have a chance to write on the chalkboard for at least one of the exercises.

These exercises are hard — harder than on the homework, quizzes, or exams. That means that you should spend some time thinking and talking about them; they're designed to be solved in groups (the best way to learn mathematics). The problems are roughly in order of increasing difficulty. I don't expect any group to solve all of them.

Don't forget to draw pictures.

1. A spring with a one-gram bob and spring constant of two grams per second^2 is placed in a viscous medium, providing a damping force with coefficient equaling two grams per second. Moreover, let's say that the spring is driven by an external force of $\cos(t/\text{sec}) - 2\sin(t/\text{sec})$ dynes (gram-centimeters per second^2).

   Let's say the spring is released from two centimeters from its equilibrium position, with an initial velocity of one centimeter per second towards its equilibrium position. Then what is the spring's velocity when it first reaches its equilibrium position?

2. A series circuit with a resistor (with resistance R), an inductor (with inductance L), a capacitor (with capacitance C), and an applied voltage $E(t)$ satisfies the differential equation
$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = E(t)$$
   where Q is the charge in the capacitor.

   (a) Let's say that all three of L, C, and R are positive, with $R^2 < 4L/C$. If $E(t)$ is zero (except for some initial applied potential), then describe how Q changes with time.

   (b) Now let's say that, except for some initial applied potential, $E(t) = \sin(kt)$ for some k. What is the long-term behavior of the circuit?

   (c) What if $R = 0$ and $E(t) = 0$?

   (d) What if $R = 0$ and $E(t) = \sin(kt)$? For what k does $|Q|$ get larger and larger (on average)?