Math 1B Worksheet 3:
Trigonometric substitutions

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Please introduce yourselves to each other, and put your names at the top of a piece of blackboard. Take turns being the scribe: each of you should have a chance to write on the chalkboard for at least one of the exercises.

These exercises are hard — harder than on the homework, quizzes, or exams. That means that you should spend some time thinking and talking about them; they're designed to be solved in groups (the best way to learn mathematics). The problems are roughly in order of increasing difficulty. I don't expect any group to solve all of them.

Don't forget to draw pictures.

1. Evaluate the integral $\displaystyle\int \frac{x^2\,dx}{\sqrt{-x^2-4x-3}}$

2. Find the area of the region bounded by the curves

   (a) $y = 2/x$ and $x + y = 4$

   (b) $y^2 - x^2 = 4$ and $y = 4\sqrt{x}$

3. Draw the ellipses and find their areas:

   (a) $25x^2 + 9y^2 - 100x + 18y - 116 = 0$

   (b) $13x^2 + 13y^2 + 10xy = 25$

4. Imagine taking a solid sphere of radius 1, and slicing it by a plane slice a distance $a < 1$ from the center. What are the volumes of the two pieces?

5. You're standing on a pier. There's a boat in the water at distance L from you, connected to a rope of length L, and you're holding the other end (the rope is completely taut). Imagine that you start to walk along the pier, pulling the rope; sketch the path the boat follows.

   In fact, the boat will follow a path called a *tractrix*; it's defined by the property that the rope is always tangent to the path of the boat. To find an equation for the path as a function $y = y(x)$, solve the following integral:

   $$\int \frac{dy}{y} = \int \frac{-\sqrt{L^2 - x^2}}{x}\,dx$$