Math 1B Worksheet 8:
Improper Integration and Arc Length

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Please introduce yourselves to each other, and put your names at the top of a piece of blackboard. Take turns being the scribe: each of you should have a chance to write on the chalkboard for at least one of the exercises.

These exercises are hard — harder than on the homework, quizzes, or exams. That means that you should spend some time thinking and talking about them; they're designed to be solved in groups (the best way to learn mathematics). I don't expect any group to solve all of them.

Don't forget to draw pictures.

1. For what (including non-integer) values of n does $\int_0^1 (\sin x)^n dx$ converge?

2. In terms of b and c, evaluate
$$\int_{-\infty}^{\infty} \frac{dx}{x^2 + bx + c}.$$

   What conditions do you need to place on b and c to assure that this integral converges?

3. Find the arc length of $y = \sqrt{x}$ as x ranges from 0 to 1.

4. Using your arc length formula, prove the formula for the circumference of a circle.