Math 1B Worksheet 9: Surface Area of a Surface of Revolution

Thursday, 20 September 2007

Theo Johnson-Freyd
http://math.berkeley.edu/~theojf/

Please introduce yourselves to each other, and put your names at the top of a piece of blackboard. Take turns being the scribe: each of you should have a chance to write on the chalkboard for at least one of the exercises.

These exercises are hard — harder than on the homework, quizzes, or exams. That means that you should spend some time thinking and talking about them; they're designed to be solved in groups (the best way to learn mathematics). The problems are roughly in order of increasing difficulty. I don't expect any group to solve all of them.

Don't forget to draw pictures.

1. Find the surface area of the figure traced out by the curve $y = \ln(x^2 + 1)$, $0 \le x \le 1$, revolved around the x-axis.

2. Find the surface area of the figure traced out by the curve $y = e^{2x}$, $0 \le x \le 1$, revolved around the x-axis.

3. Let f be a positive, continuous function on $[0, 1]$, and let $\mathcal{A}[f]$ be the surface area of the surface of revolution formed by revolving the curve $y = f(x)$, $0 \le x \le 1$ around the x axis. Let $\ell[f]$ be the length of the curve $y = f(x)$ from $x = 0$ to $x = 1$. Why is the following equation true?
$$\mathcal{A}[f + 1] = \mathcal{A}[f] + 2\pi\,\ell[f]$$

4. Say that f is a bounded, positive function on the real line such that f' is bounded. (This means that there exist real numbers M and K such that for every real number x, both $0 < f(x) < M$ and $|f'(x)| < K$ are true.) Moreover, let's say that $\int_{-\infty}^{\infty} f(x)\,dx$ converges. Now rotate the curve $y = f(x)$ around the x-axis. What can you say about whether the surface area of this surface of revolution is finite or infinite? (I.e. does it converge or diverge?) What about the volume?