

Math 32 Discussion Problems

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<http://math.berkeley.edu/~theo/f/08Fall32/>

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Most days, we will have discussion problems based on the previous day's lecture. But today, with no lecture to respond to, we get more fun exercises. These problems are from <http://ekhad.net/funmath.html>. These are designed to be solved in groups: talk about them with each other, and also with the GSI and UGAs. Don't expect to get to all of them. Indeed, feel free to go through and find the ones that you like the most.

1. You're on a high bridge one mile long. In fact you are $\frac{3}{8}$ of the way across. If your life depends on it, you can sprint at 10 miles per hour (mph). Indeed your life depends on it, because you look behind you and see a train coming. If you run back to the beginning of the bridge you'll jump out of the way just in time - just as the train reaches the bridge. If you keep going across the bridge you'll reach the end of the bridge just in time, just as the train reaches the end of the bridge. How fast is the train going?
2. Horses cost \$10, pigs cost \$3, and rabbits are only \$0.50. A farmer buys 100 animals for \$100, How many of each animal did he buy? There are two correct answers.
There are many variations floating around. Another one sets horses at \$5, pigs at \$2, and rabbits at 10 cents a piece. This configuration has only one answer.
3. You're standing on the planet Earth, which we will assume is a perfect sphere. You walk south 1 mile, east 1 mile, and north 1 mile, and you wind up exactly where you started. Where are you? For extra credit, where else might you be? There are actually many possible locations; see if you can describe them all.
4. How many ways can you arrange 6 different books, left to right, on a shelf?
5. How many numbers from 1 to 1 million are not perfect squares or perfect cubes?
6. You have a glass of water and a glass of wine. Take a spoonful of water from the water glass and add it to the wine. Stir the wine until the water is well mixed throughout. Now take a spoonful of liquid from the wine glass and pour it back into the water glass. Is there more water in the wine, or more wine in the water? Or are the ratios the same?
7. A school has 1,000 students and 1,000 lockers, all in a row. They all start out closed. The first student walks down the line and opens each one. The second student closes

the even numbered lockers. The third student approaches every third locker and changes its state. If it was open he closes it; if it was closed he opens it. The fourth student does the same to every fourth locker, and so on through 1,000 students. To illustrate, the tenth locker is opened by the first student, closed by the second, reopened by the fifth, and then closed by the tenth. All the other students pass by the tenth locker, so it winds up being closed. How many lockers are open?

8. A checkerboard is missing two of its squares. The lower left square and the upper right square are gone. That leaves 62 squares on the board. You are given 31 dominoes. Each domino is a rectangle that can cover exactly two squares on the checkerboard. Can you place all 31 dominoes on the checkerboard, so that all 62 squares are covered?
9. Join three unit squares together in a row and place another square above the middle square. This shape of area 4 is called a tetromino. Can you cover a 10 by 10 square (area 100) with 25 tetrominoes?
10. There are six people at a party. Prove that there are three people in this party who know each other, or three people who are complete strangers to each other.
11. Arrange the numbers 1 through 9 on a tic-tac-toe board, so that every row and every column and every diagonal adds up to the same number. Show that there is only one way (up to rotations and reflections) to do this.
12. Suppose you are given a list of all the real numbers between 0 and 1. It's an infinite list of course, but it's still a sequential list. For example, it might begin 0.635, 0.1314, $1/7$, $\sqrt{0.3}$, $\pi/5$, \dots . Explain how to construct a real number that is not on the list.
13. Two trains are traveling at 50 miles an hour. They are 100 miles apart, and headed straight towards each other. A fly can travel at 70 miles an hour. He starts at one train and flies towards the other. When he reaches the second train he turns around and heads back to the first. He keeps flying back and forth until the trains meet. How far has the fly traveled?
14. Three kids each contribute \$10 to purchase a \$30 cd player. (This is in Oregon; no sales tax.) After they leave, the store owner realizes the item is marked down to \$25. He hands five singles to his clerk and tells him to run out and reimburse the kids. The clerk catches up with the kids, but he is less than honest, and besides, you can't divide 5 by 3. So he gives them 3 dollars and pockets the remaining 2. Now each kid has paid \$9, for a total of \$27, and the clerk has 2, making \$29. We started out with \$30, what happened to the last dollar?