Math 32 Discussion Problems

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In groups, solve and talk about the following problems. Be sure to take turns writing on the chalkboards. Some of these problems area easy: if everyone in your group feels very confident with a type of problem, then there's no reason to work on them, but do take the time to explain the exercises to each other. Some of these problems are hard, so you should not expect to solve all of them.

Set notation

1. Use set notation to describe the following region:

2. Draw a Venn diagram to describe the relationships between the sets $A = \{x : x \text{ is a country}\}$, $B = \{x : x \text{ is a place}\}$, and $C = \{x : x \text{ is in Europe}\}$. Come up with examples of elements of each region of your Venn diagram.

3. "Simplify" the set
$$\{x \in \mathbb{R} : (x \leq 3 \text{ and } x < 5) \text{ or } x \leq 2\}.$$
Also write this set in interval notation.

4. "Simplify" the set
$$\{x \in \mathbb{R} : (x \geq 3 \text{ and } x > 5) \text{ or } x \leq 2\}.$$
Also write this set in interval notation.

5. "Simplify" the set
$$\{x \in \mathbb{R} : (x \leq 3 \text{ and } x < 5) \text{ or } x \geq 2\}.$$
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6. "Simplify" the set
$$\{x \in \mathbb{R} : (x \geq 3 \text{ and } x < 5) \text{ or } x \leq 2\}.$$
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7. "Simplify" the set
$$\{x \in \mathbb{R} : (x \leq 3 \text{ and } x > 5) \text{ or } x \leq 2\}.$$
Also write this set in interval notation.

Absolute Value

1. Solve the following equations or inequalities for x. Use whatever method you like: algebraic manipulation, graphing, guessing and checking, dots.

   (a) $|x-3|+7=12$

   (b) $12=|4x+24|$

   (c) $-2\,|x/2+3|-4>-10$

   (d) $2|x+4|\leq 16$

   (e) $7|2x-2|-3>53$

   (f) $|2x-2|\geq x+1$

   (g) $x-|2x-1|=7$

   (h) $x+|x+1|=1$

   (i) $|x^2-4|=x+2$

   For more practice with problems like these, try
   http://www.algebralab.org/practice/practice.aspx?file=Algebra2_1-7.xml

2. Prove the triangle inequality, that $|x+y|\leq|x|+|y|$ for any real numbers x and y. Here's one possible outline: $a\leq|a|$, so $x+y\leq|x|+|y|$, and similarly $-(x+y)\leq|x|+|y|$. You'll need to do some work to fill in all the steps here.

3. Show that for all real numbers a and b, we have

$$|a|-|b|\leq|a-b|$$

   Hint: start with the identity $a=(a-b)+b$, take absolute values of each side, and use the triangle inequality that $|x+y|\leq|x|+|y|$ for any real numbers x and y.

4. Show that for all real numbers a, b, and c, we have

$$|a+b+c|\leq|a|+|b|+|c|$$

   For what values of a, b, and c is this an equality?

5. Prove the following formula:

$$\max(a,b)=\frac{a+b+|a-b|}{2}$$

   Find a similar formula for $\min(a,b)$.

Solving Equations (Review and Preview)

1. Determine whether the given value is a solution to the equation:

   (a) $\frac{1}{x}=\frac{3}{x}-1$; $x=2$

   (b) $y+3+\frac{2}{y-1}=\frac{2y}{y-1}$; $y=1$

   (c) $x^2-2x-4=0$; $x=1+\sqrt{5}$ or $1-\sqrt{5}$

2. Solve each equation:

   (a) $2m-1+3m+5=6m-8$

   (b) $(x+2)(x+1)=x^2+11$

   (c) $y+3+\frac{2}{y-1}=\frac{2y}{y-1}$