

# Math 32 Discussion Problems

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<http://math.berkeley.edu/~theo/f/08Fall32/>

Tuesday 30<sup>th</sup> September, 2008

*In groups, solve and talk about the following problems. Be sure to take turns writing on the chalkboards. Some of these problems are easy: if everyone in your group feels very confident with a type of problem, then there's no reason to work on them, but do take the time to explain the exercises to each other. Some of these problems are hard, so you should not expect to solve all of them.*

## Polynomial and Rational Functions

- What is the definition of a polynomial functions? Of a power function? Give examples of functions which are polynomial, power, both, and neither.
  - Is the sum of two polynomial functions necessarily a polynomial? Why or why not? Is the product of two polynomial functions necessarily a polynomial? Why or why not? How about the difference or ratio?
- For each of the following functions, determine the  $x$ - and  $y$ -intercepts, the behavior of the function near infinity, and the behavior of the function near each intercept. Use this data to graph sketch a graph of each function.
  - $y = (x - 3)(x + 2)(x + 1)$
  - $y = (x - 3)(x - 2)(x + 1)$
  - $y = (x - 1)(x - 4)^2$
  - $y = (x - 1)^2(x - 4)^2$
  - $y = 4(x - 2)^2(x + 2)^2$
  - $y = -3x^3(x + 1)^4$
  - $y = x^3 - 9x$
  - $y = x^3 - 5x^2 - x + 5$
- What is a rational function? Give an example of a rational function that has no vertical asymptotes. Give an example of a rational function with exactly two vertical asymptotes.
  - Is the sum of two rational functions necessarily rational? Why or why not? How about the product, difference, and ratio of rational functions?
- Sketch a graph of each of the following functions, specifying the intercepts and asymptotes.
  - $y = -1/(x + 4)$
  - $y = -3/(x + 2)$
  - $y = (x - 1)/(x + 1)$
  - $y = 2x/(x + 3)$
  - $y = (3x + 2)/(x - 3)$
  - $y = -1/(x - 2)^2$
  - $y = -3/(x + 1)^2$
  - $y = -1/(x + 2)^3$
  - $y = x/[(x + 1)(x - 3)]$
- Find all horizontal and vertical intercepts and asymptotes for the following functions. Also find where the functions cross their horizontal asymptotes.
  - $y = \frac{(x - 4)(x + 2)}{(x - 1)(x - 3)}$
  - $y = \frac{(x - 1)(x - 3)}{(x + 1)^2}$
  - $y = \frac{(x + 1)^2}{(x - 1)(x - 3)}$
- Sketch a graph of  $y = x^2/(x + 2)$ . Hint: show that  $y = x - 2$  is a slant asymptote.