Math 32 Discussion Problems

GSI: Theo Johnson-Freyd
http://math.berkeley.edu/~theojf/08Fall32/

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Your book carefully uses "$\ln x$" for the "natural logarithm" $\log_e x$, and you are probably used to writing "$\log x$" without a subscript to mean the "common logarithm" $\log_1 0x$. In fact, most mathematicians never use common logs (but chemists and engineers do), and write (and say) "$\log x$" when they mean "$\ln x$". I will try to be careful in this class, at least in the handouts.

Exponential and Logarithmic Functions

1. Solve the equation $\log_2 x = \log_x 2$. Solve the equation $\log_2 x = \log_x 3$.

2. Solve each equation or inequality:
   (a) $x^{1+\log_x 16} = 4x^2$
   (b) $3\log_{10}(4x+3) < 1$
   (c) $\log_{\sqrt{x}}(\sqrt{x+4}+2) = 2$
   (d) $10^{-x^2} \leq 10^{-12}$
   (e) $\frac{2}{3}(1-e^{-x}) \leq -3$
   (f) $\ln \frac{3x-2}{4x+1} > \ln 4$

3. Find both solutions to the equation $x^{(x^x)} = (x^x)^x$.

4. Let $f(x) = \ln(x+\sqrt{x^2+1})$. Find $f^{-1}(x)$.

5. A bank pays 7% interest compounded annually. What principal will grow to \$10 000 in 10 years?

6. A sum of \$3000 is placed in a savings account at 6% per annum. How much is in the account after 1 year if the interest is compounded annually? semiannually? daily?

7. Given a nominal rate of 6% per annum, compute the effective rate under continuous compounding of interest.

8. Which is the better investment: 5% compounded annually, or 4% compounded continuously?

9. How long will it take an investment to double if it's invested at a rate of 7% compounded annually?

10. One account has a \$1000 principal, compounded continuously at 5% per annum. Another has a \$500 principal, compounded continuously at 10% per annum. How long will it be until the second account has more money in it than the first?

11. (a) Last week we showed that e^x was much bigger than any polynomial function. Using the fact that $e^x \gg x$, show that $\ln x \ll x$.
    (b) Use the fact that for each n eventually $e^x > x^n$ to show that for each n eventually $\ln x < x^{1/n}$.

    (Incidentally, the notation "$f(x) \gg g(x)$" means that "for each real number C, eventually $f(x) > Cg(x)$".)

12. Prove that $\log_2 3$ is irrational. Hint: First show that $\log_2 3$ is positive. Then assume that it is rational — that $\log_2 3 = m/n$ where m and n are positive integers — and conclude that some (positive integer) power of 2 is equal to a (positive integer) power of 3 — find these powers in terms of m and n. But every power of 2 is even, and every power of 3 is odd, so conclude that your assumption must have been false.