Math 32 Discussion Problems

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Tuesday 21st October, 2008

Your book carefully uses "$\ln x$" for the "natural logarithm" $\log_e x$, and you are probably used to writing "$\log x$" without a subscript to mean the "common logarithm" $\log_1 0x$. In fact, most mathematicians never use common logs (but chemists and engineers do), and write (and say) "$\log x$" when they mean "$\ln x$". I will try to be careful in this class, at least in the handouts.

Trigonometry

1. Let ABC be a right triangle with C the right angle. If $AB = 3$ and $BC = 1$, compute the six trigonometric functions of angle B.

2. You should memorize, if you haven't already, the following values of sin, cos, and tan:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
$30°$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
$45°$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$60°$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$

   Use these values to compute the following:

   (a) $\cos 60° + 2\sin^2 30°$

   (b) $\sin 30° \cos 60° + \cos 30° \sin 60°$

   (c) $2 \sin 45° \cos 45°$

   (d) $\csc^2 60° - \cot^2 60°$

   (e) $\sqrt{(1 + \cos 60°)/2}$

   (f) $(1 - \cos 60°)/\sin 60°$

3. Simplify the following expressions:

   (a) $10 \sin\theta\cos\theta + 4\sin\theta\cos\theta - 16\sin\theta\cos\theta$

   (b) $-\cos^2\theta\sin^2\theta + (2\sin\theta\cos\theta)^2$

   (c) $(3 - 2\tan\theta)^2$

   (d) $(\sec^2\theta - 3)(\sec^2\theta + 3)$

   (e) $(5 - 2\tan\theta)/(2\tan\theta - 5)$

   (f) $(1/\sin\theta) - (3/\cos\theta)$

4. Use the given information to find the remaining five trigonometric angles, assuming all angles are acute.

   (a) $\sin\theta = 2/5$

   (b) $\cos\theta = \sqrt{7}/3$

   (c) $\cos A = 8/17$

   (d) $\tan B = 5$

   (e) $\csc C = \sqrt{5}/2$

   (f) $\cot\alpha = \sqrt{3}/2$

5. Simplify each expression, writing in terms of sin and cos:

(a) $\dfrac{\sin^4 A - \cos^4 A}{\sin A - \cos A}$

(b) $\sin\theta \csc\theta \tan\theta$

(c) $\dfrac{\cos A - 2\sin A\cos A}{\cos^2 A - \sin^2 A + \sin A - 1}$

(d) $\dfrac{3\sin\theta + 6}{\sin^2\theta - 4}$

(e) $\cot\theta + \dfrac{1 - 2\cos^2\theta}{\sin\theta\cos\theta}$

(f) $(\sec A + \tan A)(\sec A - \tan A)$

(g) $\dfrac{\tan\theta + \tan\theta\sin\theta - \cos\theta\sin\theta}{\sin\theta\tan\theta}$

(h) $\dfrac{\cos(90^\circ - A)}{\csc A} + \dfrac{\sin(90^\circ - A)}{\sec A}$

(i) $\dfrac{\cos A\tan A}{\tan(90^\circ - A)} - \dfrac{1}{\sin(90^\circ - A)}$

(j) $\sin^2(90^\circ - \beta) + \cos^2(90^\circ - \beta)$

6. Suppose that $A\sin\theta + \cos\theta = B\sin\theta - \cos\theta = 1$. Show that $AB = 1$.

7. If $a = \sin\theta + \cos\theta$ and $b = \sin\theta - \cos\theta$, show that

$$\tan\theta = \frac{a+b}{a-b}$$

8. Suppose that β is an acute angle and

$$\sin\beta = \frac{m^2 - n^2}{m^2 + n^2}$$

for $m > n > 0$. Show that

$$\cos\beta = \frac{2mn}{m^2 + n^2} \quad \text{and} \quad \tan\beta = \frac{m^2 - n^2}{2mn}$$

9. From a point level with and 1000 ft away from the base of Washington Monument, the angle of elevation to the top of the monument is just shy of 30°. Estimate the height of the monument.

10. A surveyor stands a distance d from the base of a building. On top of the building is a vertical radio antenna. Let α denote the angle of elevation when the surveyor sights to the top of the building, and β the angle of elevation to the top of the antenna. Express the length of the antenna in terms of α, β, and d.

11. Recall that a triangle with side a and b and angle between them θ has area $A = \frac{1}{2}ab\sin\theta$. Find the area of the regular pentagon inscribed in a unit circle (in terms of sin). How about the area of the regular pentagon circumscribed around a unit circle? How about the areas of the regular n-gons inscribed and circumscribed about a unit circle?

12. A vertical tower of height h stands on level ground. From a point P at ground level and due south of the tower, the angle of elevation to the top of the tower is α. From a point Q at ground level and due west of the tower, the angle of elevation to the top of the tower is β. If d is the distance between P and Q, show that

$$h = \frac{d}{\sqrt{\cot^2\alpha + \cot^2\beta}}$$