Math 32 Discussion Problems

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The textbook introduces *radians* — the better units (than degrees) for measuring angles — in section 7.1. So until then I will try to use degrees consistently on the handouts. But, for the record, the conversion is that $1^\circ = \pi/180$, so for example $30^\circ = 30 \times \pi/180 = \pi/6$.

Trigonometry

1. From a point level with and 1000 ft away from the base of Washington Monument, the angle of elevation to the top of the monument is just shy of 30°. Estimate the height of the monument.

2. A surveyor stands a distance d from the base of a building. On top of the building is a vertical radio antenna. Let α denote the angle of elevation when the surveyor sights to the top of the building, and β the angle of elevation to the top of the antenna. Express the length of the antenna in terms of α, β, and d.

3. Evaluate the six trigonometric functions of the following angles:
   (a) 270° (b) -270° (c) 450° (d) -720°

4. Which is larger?
   (a) $\sin 60^\circ$ or $\sin 70^\circ$ (c) $\sin 160^\circ$ or $\sin 170^\circ$ (e) $\sin 230^\circ$ or $\sin 320^\circ$
   (b) $\sin 190^\circ$ or $\cos 190^\circ$ (d) $\sin 190^\circ$ or $\sin(-190^\circ)$ (f) $\sin(-230^\circ)$ or $\cos(-230^\circ)$

5. The area of a triangle with two sides a and b and included angle θ is $\frac{1}{2}ab\sin\theta$. Use this to determine the areas of the following triangles:
   (a) Two of the sides are 5 cm and 7 cm, and the angle between those sides is 120°.
   (b) Two of the sides are 3 m and 6 m, and the included angle is 150°.
   (c) Two of the sides are 1 in and 2 in, and the included angle is 60°. What is this triangle?

6. Use the above formula for area to prove that the area of any quadrilateral is one-half the product of the diagonals times the sine of the angle of intersection of the diagonals.

7. For which values of θ are each of the following equations valid:
$$\log_2(\sin^2\theta) = 2\log_2(\sin\theta) \tag{1}$$
$$\log_{10}(\tan\theta) = -\log_{10}(\cot\theta) \tag{2}$$

8. Use the properties of logarithms, and the Pythagorean identity, to prove that
$$\ln\sqrt{1-\sin\theta} + \ln\sqrt{1+\sin\theta} = \ln\cos\theta \tag{3}$$
whenever both sides are defined. What are the domains of the right- and left-hand sides of the above equation?

9. Use the given information, and the Pythagorean identity, to determine the remaining five trigonometric values:
   (a) $\cos\theta = -3/4$, $90° < \theta < 180°$.
   (b) $\sin\theta = \sqrt{3}/6$, $90° < \theta < 180°$.
   (c) $\csc A = -3$, $270° < A < 360°$
   (d) $\csc A = 6/5$, $90° < A < 180°$
   (e) $\sec B = -3/2$, $180° < B < 270°$
   (f) $\sec B = 25/24$, $270° < B < 360°$

10. One of the two equations is an identity, and the other is not. Decide which, prove the equation that is an identity, and give an example showing the other equation is not.

$$(\csc\beta - \cot\beta)^2 = \frac{1+\cos\beta}{1-\cos\beta} \tag{4}$$

$$\cot\beta + \frac{\sin\beta}{1+\cos\beta} = \csc\beta \tag{5}$$

11. Prove the following equations are identities:

$$\frac{2\sin^3\beta}{1-\cos\beta} = 2\sin\beta + 2\sin\beta\cos\beta \tag{6}$$

$$\frac{\sec\theta - \csc\theta}{\sec\theta + \csc\theta} = \frac{\tan\theta - 1}{\tan\theta + 1} \tag{7}$$

$$1 - \frac{\sin^2\theta}{1+\cot\theta} - \frac{\cos^2\theta}{1+\tan\theta} = \sin\theta\cos\theta \tag{8}$$

$$(\sin^2\theta)(1 + n\cot^2\theta) = (\cos^2\theta)(n + \tan^2\theta) \tag{9}$$

$$(r\sin\theta\cos\phi)^2 + (r\sin\theta\sin\phi)^2 + (r\cos\theta)^2 = r^2 \tag{10}$$

12. If $\tan\alpha\tan\beta = 1$ and α and β are acute, show that $\sec\alpha = \csc\beta$.