Math 32 Discussion Problems

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Recall the conversion between degrees and radians: $1^\circ = \pi/180$, so for example $30^\circ = 30 \times \pi/180 = \pi/6$. Areas and lengths of circular pieces are easy in radians: an arc of angle θ (radians) in a circle with radius r has length θr, and the corresponding wedge of the circle has area $\theta r^2/2$.

Radians

1. If you didn't do them last time, prove that the following equations are identities:
   (a) $\dfrac{2\sin^3\beta}{1-\cos\beta} = 2\sin\beta + 2\sin\beta\cos\beta$
   (b) $\dfrac{\sec\theta - \csc\theta}{\sec\theta + \csc\theta} = \dfrac{\tan\theta - 1}{\tan\theta + 1}$
   (c) $1 - \dfrac{\sin^2\theta}{1+\cot\theta} - \dfrac{\cos^2\theta}{1+\tan\theta} = \sin\theta\cos\theta$
   (d) $(\sin^2\theta)(1 + n\cot^2\theta) = (\cos^2\theta)(n + \tan^2\theta)$
   (e) $(r\sin\theta\cos\phi)^2 + (r\sin\theta\sin\phi)^2 + (r\cos\theta)^2 = r^2$
2. Evaluate the six trigonometric functions of the following numbers:
   (a) π (c) $\pi/2$ (e) $\pi/6$ (i) $3\pi/4$
   (b) 2π (d) $-\pi/2$ (f) $\pi/3$ (j) $-7\pi/6$
3. Which is larger?
   (a) $\sin 1$ or $\sin 2$ (c) $\sin 9$ or $\sin 3\pi$ (e) $\sin(-1)$ or $\sin 315^\circ$
   (b) $\sin 1$ or $\cos 1$ (d) $\sin 3$ or $\sin(-3)$ (f) $\sin 10$ or $\cos 10$
4. Are there any real numbers with the property that x degrees equals x radians? How about x degrees equaling $2x$ radians?
5. Consider a circle with radius r and a central angle of value θ. Find the length of the arc subtended by the angle, and also the area of the corresponding sector of the circle, for the following values of r and θ:
   (a) $r = 4$ cm; $\theta = \pi/10$ (b) $r = 16$ m; $\theta = 5^\circ$ (c) $r = 21$ ft; $\theta = 170^\circ$
6. A wheel with radius 20 cm turns at 15 revolutions per second. Since 1 revolution is 2π radians, what is the angular speed of the wheel in radians per second? Since 1 radian in a circle of radius r corresponds to r units of arclength, find the linear speed of a point on the circumference of the wheel. If the wheel (radius 20 cm, turning at 15 revolutions per second) is set on the ground, how fast will the axel move relative to the ground?
7. A wheel 3 ft in diameter makes x revolutions as it rolls along the ground. Find x, given that the distance traveled by the center of the wheel is 22619 ft.

8. Two wheels turning in place are driven by a belt. If the wheels have radius r and R, and the wheel of radius r turns at an angular speed α, find the angular speed of the wheel of radius R.

9. Suppose that in $\triangle ABC$, angle C is a right angle. Let a, b, and c be the lengths of the sides opposite angles A, B, and C respectively. Prove that:

$$a^2 \cos^2 B - b^2 \cos^2 A = a^2 - b^2$$

10. A half-circle centered at O with diameter $\overline{AB}$ is divided by a radius $\overline{OC}$, where $\angle BOC = \theta$. Find the areas of the two sectors of the circle in terms of θ and the radius r. Find the product of the two areas, and find the value of θ that maximizes this product.

11. Consider a circle with radius 1, center O. Let $\overline{AE}$ be a diameter, C a point on the circle with $\angle COE = \theta$. Two semicircles, with diameters $\overline{AC}$ and $\overline{CE}$, bulge out of the circle. What is the area of $\triangle ACE$? What is the combined area of the two lunes — the parts of the circles with diameters $\overline{AC}$ and $\overline{CE}$ that are not in the big circle?