Math 32 Discussion Problems

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Thursday 30th October, 2008

More on trig functions

1. A half-circle centered at O with diameter $\overline{AB}$ is divided by a radius $\overline{OC}$, where $\angle BOC = \theta$. Find the areas of the two sectors of the circle in terms of θ and the radius r. Find the product of the two areas, and find the value of θ that maximizes this product.

2. If $\cot\theta = -1/\sqrt{3}$ and $\cos\theta < 0$, compute $\csc\theta$ and $\sin\theta$.

3. Evaluate the following trigonometric functions, using the fact that $\cos t = \cos(t + 2\pi)$ and $\sin t = \sin(t + 2\pi)$:

   (a) $\sin(17\pi/4)$ (c) $\cos 11\pi$ (e) $\tan(-7\pi/4)$ (g) $\sec(\frac{11\pi}{6} + 2\pi)$

   (b) $\sin(-17\pi/4)$ (d) $\cos(53\pi/4)$ (f) $\cos(7\pi/4)$ (h) $\csc(2\pi - \frac{\pi}{3})$

4. Prove the following identities:

   (a) $\sin^2 t - \cos^2 t = \dfrac{1 - \cot^2 t}{1 + \cot^2 t}$

   (b) $\dfrac{1 + \tan s}{1 - \tan s} = \dfrac{\sec^2 s + 2\tan s}{2 - \sec^2 s}$

   (c) $(\tan\theta)(1 - \cot^2\theta) + (\cot\theta)(1 - \tan^2\theta) = 0$

   (d) $\cot\theta + \tan\theta + 1 = \dfrac{\cot\theta}{1 - \tan\theta} + \dfrac{\tan\theta}{1 - \cot\theta}$

5. Show that $\sin\theta\cos\theta \le 1/2$ for every θ. For what θ values is this an equality? Hint: use the fact that $\sqrt{ab} \le (a + b)/2$ when a and b are positive real numbers, with equality only when $a = b$, with $a = \sin^2\theta$ and $b = \cos^2\theta$. Then use the fact that $x \le |x|$ for any real number x.

6. (a) What is the period of the function $\sin x$ for x a real number?
   (b) What is the period of the function $\sin(2x)$? $\sin(\pi x)$? Hint: the period of x is the smallest positive number p so that $f(x) = f(x+p)$ for every x. We know that $\sin y = \sin(y+2\pi)$, so plug in $x + p$ for x, and solve for p.
   (c) What are all the zeros of the sine function?
   (d) For what values of x is $\sin x$ increasing? Decreasing?

7. Graph $\sin^{-1}(\sin x)$, where $\sin^{-1}$ is the inverse-sine function, defined as having domain $x \in [-1, 1]$ and outputting the unique number $y \in [-\pi, \pi]$ such that $x = \sin y$.

8. If x_0 is one solution to $\sin x = a$, which of the following are also solutions?

   (a) $x_0 + 2\pi$ (c) $x_0 - 2\pi$ (c) $x_0 - \pi$ (d) $x_0 + 6\pi$

   (b) $x_0 + \pi$ (d) $2\pi - x_0$ (d) $\pi - x_0$ (e) $5\pi - x_0$