Math 32 Discussion Problems

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More on trig functions

1. (a) What is the period of the function $\sin x$ for x a real number?
   (b) What is the period of the function $\sin(2x)$? $\sin(\pi x)$? Hint: the period of x is the smallest positive number p so that $f(x) = f(x+p)$ for every x. We know that $\sin y = \sin(y+2\pi)$, so plug in $x+p$ for x, and solve for p.
   (c) What are all the zeros of the sine function?
   (d) For what values of x is $\sin x$ increasing? Decreasing?

2. If x_0 is one solution to $\sin x = a$, which of the following are also solutions?

   (a) $x_0 + 2\pi$ (c) $x_0 - 2\pi$ (c) $x_0 - \pi$ (d) $x_0 + 6\pi$

   (b) $x_0 + \pi$ (d) $2\pi - x_0$ (d) $\pi - x_0$ (e) $5\pi - x_0$

3. Let $\sin^{-1}$ is the inverse-sine function, defined as having domain $x \in [-1, 1]$ and outputting the unique number $y \in [-\pi/2, \pi/2]$ such that $x = \sin y$. Let $\cos^{-1}$ be the inverse-cosine function, with domain $x \in [-1, 1]$ and outputting the unique number $y \in [0, \pi]$ such that $x = \cos y$. Graph the following functions:

   (a) $\sin^{-1}(\sin x)$ (b) $\sin(\sin^{-1} x)$ (c) $\cos^{-1}(\cos x)$ (d) $\cos(\cos^{-1} x)$

4. Let $\tan^{-1}$ be the inverse-tangent function, defined as outputting the unique $y \in [-\pi/2, \pi/2]$ such that $x = \tan y$.

   (a) What is the domain of $\tan^{-1}$? What is the range?
   (b) Graph $\tan^{-1}(\tan x)$ and $\tan(\tan^{-1} x)$.
   (c) Show that $\cot(\tan^{-1} x) = 1/x$ for every $x \neq 0$.

5. (a) Show that if $x \in [-1, 1]$, then $\cos(\sin^{-1} x) = \sqrt{1 - x^2} = \sin(\cos^{-1} x)$.
   (b) Evaluate $\tan(\sin^{-1} x)$ and $\tan(\cos^{-1} x)$.
   (c) Find $\sin(\tan^{-1} x)$ and $\cos(\tan^{-1} x)$.

6. Graph the following functions. Indicate the x-intercepts and the coordinates of the highest and lowest points.

   (a) $\sin(x - \pi/6)$ (c) $\sin(2x - \pi/2)$ (c) $3\sin(x/2 + \pi/6)$ (d) $\frac{1}{2}\sin(\pi x/2 - \pi^2)$

   (b) $-\cos(x + \pi/4)$ (d) $\cos(2x - \pi)$ (d) $4\cos(3x - \pi/4)$ (e) $1 - \cos(2x - \pi/3)$

7. Show that $\sin\theta\cos\theta \leq 1/2$ for every θ. For what θ values is this an equality? Hint: use the fact that $\sqrt{ab} \leq (a + b)/2$ when a and b are positive real numbers, with equality only when $a = b$, with $a = \sin^2\theta$ and $b = \cos^2\theta$. Then use the fact that $x \leq |x|$ for any real number x.

8. (a) Let $f(x) = \sin x \cos x$. It's a fact, supported by plotting points, that $f(x)$ is sinosoidal. Find $f(0)$, $f(\pi/6)$, $f(\pi/4)$, $f(\pi/3)$, $f(\pi/2)$, $f(3\pi/4)$, and $f(\pi)$, and graph these points. Use this and the above function to guess the amplitude, frequency, and phase shift of $f(x)$; i.e. find constants A, B, and C based on your graph that make $f(x) = A\sin(Bx - C)$.

   (b) The function $g(x) = \sin^2 x$ is also sinosoidal. Plot points $g(0)$, $g(\pi/6)$, $g(\pi/4)$, $g(\pi/3)$, $g(\pi/2)$, $g(3\pi/4)$, and $g(\pi)$, etc., until you have enough data to guess the amplitude, frequency, phase shift, and vertical translation, so that you can write $g(x) = A\sin(Bx - C) + D$. Since $\sin^2 x = 1 - \cos^2 x$, use your answer to find the amplitude, frequency, phase shift, and vertical translation for $h(x) = \cos^2 x$, and check your answer by plotting points.

9. Graph each function, specifying the intercepts and asymptotes. Hint: any problem about sec and csc is really about cos and sin.

   (a) $\sec x$ (c) $\csc(x - \pi/6)$ (c) $-\frac{1}{2}\csc(2\pi x)$ (d) $\sec(x+1)$

   (b) $\csc x$ (d) $2\csc x$ (d) $-2\sec x$ (e) $-2\sec(\pi x/3)$

10. When we defined the inverse sine, cosine, and tangent) functions, we had to limit the domains of sine, cosine, and tangent. There was always a best choice for how to limit the domain: we always chose the largest continuous domain of sine, cosine, or tangent that contained 0 on which the function was one-to-one.

    (a) Does this method work to define the secant-inverse function $\sec^{-1}$? Graph $y = f(x) = \cos^{-1}(1/x)$ and specify the domain and range. Prove that $f(x)$ is an inverse function to $\sec(x)$, by calculating $f(\sec(x))$ and $\sec(f(x))$.

    (b) Does this method work to define the cosecant-inverse function $\csc^{-1}$? Graph $y = g(x) = \csc^{-1}(1/x)$ and specify the domain and range. Prove that $g(x)$ is an inverse function to $\csc(x)$, by calculating $g(\csc(x))$ and $\csc(g(x))$.