Math 32 Discussion Problems

GSI: Theo Johnson-Freyd
http://math.berkeley.edu/∼theojf/08Fall32/

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Trig review

1. What are *radians*? How are they defined? How do you convert between radians and degrees?
2. What is the formula for the length of an arc subtended by a given angle? What's the formula for the area of a sector of a circle?
3. Find the area of the shaded region as a function of θ and r:

4. Plot the following radian angle measures on the unit circle. Label each point with the sine and cosine of the angle.
   (a) $2\pi/3$ (c) $-\pi/6$ (c) $-5\pi/4$ (d) $5\pi/3$
   (b) $-\pi/2$ (d) $13\pi/4$ (d) $9\pi/4$ (e) $-7\pi/4$
5. In the equation $x^4 + 6x^2y^2 + y^4 = 32$, make the substitutions
   $$x = X\cos\frac{\pi}{4} - Y\sin\frac{\pi}{4} \quad \text{and} \quad y = X\sin\frac{\pi}{4} + Y\cos\frac{\pi}{4}$$
   and show that the result simplifies to $X^4 + Y^4 = 16$.
6. (a) What is the definition of the *amplitude* of a sinosoidal wave? What is the *period*? Draw a graph of a sinosoidal wave with period 3 and amplitude 4.
   (b) What is the amplitude of $y = \sin x$? What is the period? What about $y = \cos x$?
7. (a) Give examples of four negative real numbers x such that $\cos x = 0$.
   (b) Give examples of four positive real numbers x such that $\sin x = 1$.
   (c) What are the four smallest positive real numbers x such that $\sin x = 1/2$?
8. Show that $\sin\theta\cos\theta \le 1/2$ for every θ. For what θ values is this an equality? Hint: use the fact that $\sqrt{ab} \le (a+b)/2$ when a and b are positive real numbers, with equality only when $a = b$, with $a = \sin^2\theta$ and $b = \cos^2\theta$. Then use the fact that $x \le |x|$ for any real number x.

9. (a) Let $f(x) = \sin x \cos x$. It's a fact, supported by plotting points, that $f(x)$ is sinosoidal. Find $f(0)$, $f(\pi/6)$, $f(\pi/4)$, $f(\pi/3)$, $f(\pi/2)$, $f(3\pi/4)$, and $f(\pi)$, and graph these points. Use this and the above function to guess the amplitude, frequency, and phase shift of $f(x)$; i.e. find constants A, B, and C based on your graph that make $f(x) = A\sin(Bx - C)$.

   (b) The function $g(x) = \sin^2 x$ is also sinosoidal. Plot points $g(0)$, $g(\pi/6)$, $g(\pi/4)$, $g(\pi/3)$, $g(\pi/2)$, $g(3\pi/4)$, and $g(\pi)$, etc., until you have enough data to guess the amplitude, frequency, phase shift, and vertical translation, so that you can write $g(x) = A\sin(Bx - C) + D$. Since $\sin^2 x = 1 - \cos^2 x$, use your answer to find the amplitude, frequency, phase shift, and vertical translation for $h(x) = \cos^2 x$, and check your answer by plotting points.

10. Graph each function, specifying the intercepts and asymptotes. Hint: any problem about sec and csc is really about cos and sin.

    (a) $\sec x$ (c) $\csc(x - \pi/6)$ (c) $-\frac{1}{2}\csc(2\pi x)$ (d) $\sec(x + 1)$

    (b) $\csc x$ (d) $2\csc x$ (d) $-2\sec x$ (e) $-2\sec(\pi x/3)$