Math 32 Discussion Problems

GSI: Theo Johnson-Freyd
http://math.berkeley.edu/~theojf/08Fall32/

Thursday 13th November, 2008

Sum, product, etc. formulas from Trigonometry

1. Simplify each expression:

   (a) $\sin(t + \frac{\pi}{6}) - \sin(t - \frac{\pi}{6})$ (b) $\cos(\theta - \frac{\pi}{4}) + \cos(\theta + \frac{\pi}{4})$

2. Use the sum-of-angles formulas to prove each each equation is an identity:

   (a) $\cos(A - B) - \cos(A + B) = 2 \sin A \sin B$ (b) $\sin(C + D) \sin(C - D) = \cos^2 C - \cos^2 B$

   Use part (a) to prove that $\sin A \sin B = \frac{1}{2}(\cos(A - B) - \cos(A + B))$. Use part (b) with $C = (A + B)/2$ and $D = (A - B)/2$ to find another formula for $\sin A \sin B$.

   Conversely, use the half-angle formula for cosine to show that your two formulas for $\sin A \sin B$ are the same.

3. Prove the following identities:

   (a) $\dfrac{\sin(A + B)}{\sin(A - B)} = \dfrac{\tan A + \tan B}{\tan A - \tan B}$ (b) $\dfrac{\cos(A + B)}{\cos(A - B)} = \dfrac{1 - \tan A \tan B}{1 + \tan A \tan B}$

4. Use the double-angle formula for cosine to show that

   $$\sin^4 \theta = \frac{3 - 4 \cos 2\theta + \cos 4\theta}{8}$$

   Find a similar formula for $\cos^4 \theta$.

5. Use the addition formulas for sine and cosine to calculate the values of the trigonometric functions at $15^\circ = \pi/12$. Thus, draw a unit circle with all 24 multiples of $15^\circ = \pi/12$ drawn in, and specify the sine and cosine at each one.

6. Use the half-angle formulas for sine and cosine to calculate the values of the trigonometric functions at $22.5^\circ = \pi/8$. Thus, draw a unit circle with all 16 multiples of $22.5^\circ = \pi/8$ drawn in, and specify the sine and cosine at each one.

7. Use the following steps to calculate the values of the trigonometric functions at $18^\circ = \pi/10$:

   (a) Prove that $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$.

   (b) Hence, provide reasons for the following equations:

   $$\sin 36^\circ = \cos 54^\circ \tag{1}$$
   $$2 \sin 18^\circ \cos 18^\circ = 4 \cos^3 18^\circ - 3 \cos 18^\circ \tag{2}$$
   $$2 \sin 18^\circ = 4(1 - \sin^2 18^\circ) - 3 \tag{3}$$

   (c) Solve equation (3) as a quadratic equation for $x = \sin 18^\circ$.

Thus, draw a unit circle with all 20 multiples of $18^\circ = \pi/10$ drawn in, and specify the sine and cosine at each one.

8. Use the fact that $3^\circ = 18^\circ - 15^\circ$ to find $\sin 3^\circ$ and $\cos 3^\circ$. Use the fact that $4.5^\circ = 22.5^\circ - 18^\circ$ to find $\sin 4.5^\circ$ and $\cos 4.5^\circ$. Hence, find $\sin 1.5^\circ$ and $\cos 1.5^\circ$.

   Incidentally, evaluating the sine and cosine of 1° is much harder. There are similar steps as in the previous problem, but they lead to a cubic equation rather than a quadratic one.

9. Understand the "picture proofs" (these are from *Proofs Without Words: Exercises in Visual Thinking* by Roger Nelson).