Math 32 Discussion Problems

GSI: Theo Johnson-Freyd
http://math.berkeley.edu/~theojf/08Fall32/

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Trigonometric Equations and Review

1. Determine whether each of the following statements is TRUE or FALSE.
   (a) The equation $\tan^2 t + 1 = \sec^2 t$ is true for every real number t.
   (b) There is no real number x satisfying the equation $\cos(\frac{\pi}{4} + x) = 2$.
   (c) There is no real number x satisfying the equation $\cos(\frac{\pi}{4} + x) = \cos x$.
   (d) For every number x in the closed interval $[-1, 1]$, we have $\sin^{-1} x = 1/\sin x$.
   (e) There is no real number x for which $\sin^{-1} x = 1/\sin x$.
   (f) The equation $\sin(x + y) = \sin x \cos y + \cos x \sin y$ holds for all real numbers x and y.
   (g) The equation $\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x\ \tan y}$ holds for all real numbers x and y.

2. Prove that the following equations are identities:
   (a) $\cot(x + y) = \dfrac{\cot x \cot y - 1}{\cot x + \cot y}$
   (b) $\sin 2x = \dfrac{2 \tan x}{1 + tan^2 x}$
   (c) $\tan^2 x - \tan^2 y = \dfrac{\sin(x + y)\sin(x - y)}{\cos^2 x \cos^2 y}$
   (d) $\tan\left(x + \dfrac{\pi}{4}\right) = \dfrac{1 + \tan x}{1 - \tan x}$
   (e) $4 \sin(x/4) \cos(x/4) \cos(x/2) = \sin x$
   (f) $\dfrac{\cot x - 1}{\cot x + 1} = \dfrac{1 - \sin 2x}{\cos 2x}$
   (g) $\tan\left(\dfrac{\pi}{4} + \dfrac{t}{2}\right) = \tan t + \sec t$
   (h) $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$
   (i) $\dfrac{\sin 3x}{\sin x} - \dfrac{\cos 3x}{\cos x} = 2$
   (j) $\cos^4 x - \sin^4 x = \cos 2x$

3. Find all real solutions of each equation. Only write your final answer using the inverse trig functions if absolutely necessary.
   (a) $\tan^2 x - 3 = 0$
   (b) $\cot^2 x - \cos x = 0$
   (c) $1 + \sin x = \cos x$
   (d) $2 \sin 3x - \sqrt{3} = 0$
   (e) $\sin x - \cos 2x + 1 = 0$
   (f) $\sin x + \sin 2x = 0$
   (g) $3 \csc x - 4 \sin x = 0$
   (h) $2 \sin^2 x + \sin x - 1 = 0$
   (i) $2 \sin^4 x - 3 \sin^2 x + 1 = 0$

4. Evaluate each expression without using a calculator:
   (a) $\cos^{-1}(-\sqrt{2}/2)$
   (b) $\sin^{-1} 0$
   (c) $\tan^{-1} \sqrt{3}$
   (d) $\cos^{-1}(\cos 5)$
   (d) $\tan^{-1}(-1)$
   (e) $\sin(\sin^{-1} 1)$
   (f) $\sin^{-1}(\sin \frac{\pi}{7})$
   (g) $\cot(\cos^{-1} \frac{1}{2})$
   (g) $\sin^{-1}(\sin 2)$
   (h) $\sin(\cos^{-1}(-\frac{1}{2}))$
   (i) $\sin(\frac{3\pi}{2} + \cos^{-1} \frac{3}{5})$
   (j) $\sin(2 \sin^{-1} \frac{4}{5})$

5. Prove each equation:

(a) $\tan(\tan^{-1} x + \tan^{-1} y) = \dfrac{x+y}{1-xy}$

(b) $\tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right) = \sin^{-1} x$

(c) $\sin(2\tan^{-1} x) = \dfrac{2x}{1+x^2}$

(d) $\cos(2\cos^{-1} x) = 2x^2 - 1$

(e) $\sin\left(\dfrac{1}{2}\sin^{-1}(x^2)\right) = \sqrt{\dfrac{1}{2} - \dfrac{1}{2}\sqrt{1-x^4}}$

(f) $\tan^{-1}\left(\dfrac{1}{3}\right) + \tan^{-1}\left(\dfrac{1}{5}\right) = \tan^{-1}\left(\dfrac{4}{7}\right)$