Math 53 Quiz 3

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Name: ____

Time (circle one): 12:10 - 1:00 3:10 - 4:00

a. (3 pts) What is the point on the line through the origin and $\langle 1,2,2\rangle$ that is closest to $\langle 1,1,0\rangle$? *Hint: one vector is projecting onto the other; draw a picture to figure out which.*

b. (3 pts) What is the equation of the plane passing through the origin and the vectors $\langle 1,2,2\rangle$ and $\langle 1,1,0\rangle$? Please write your answer in the form "all vectors $\vec{r}$ such that $\vec{r}\cdot\vec{n}=0$" for some $\vec{n}$ (i.e., find this $\vec{n}$).

c. (4 pts) What is the image of $\langle 1,1,0\rangle$ under a $90°$ rotation around the origin in the plane in the previous problem (i.e., around the line generated by the vector $\vec{n}$)? *Hint: be sure to think about the lengths of these vectors.*

   You may rotate in the clockwise or counterclockwise direction, I don't care. If your answer to the previous question is wrong, you can still get full marks for this part if your method is correct, but be sure to show enough work that I can tell.

Please use extra paper as necessary. For each part, partial credit will be assigned based on correct work (you do need to show some work, enough so that I know how you solved the problem). Please simplify and $\boxed{\text{box}}$ your answers.

a. We project the vector $\langle 1,1,0\rangle$ onto the line generated by $\langle 1,2,2\rangle$ to get

$$\frac{\langle 1,1,0\rangle\cdot\langle 1,2,2\rangle}{\langle 1,2,2\rangle^2}\langle 1,2,2\rangle = \frac{3}{9}\langle 1,2,2\rangle = \boxed{\langle \frac{1}{3},\frac{2}{3},\frac{2}{3}\rangle}$$

b. By taking the cross-product, we can find a vector perpendicular to the plane (any non-zero multiple of this vector also works):

$$\vec{n} = \langle 1,1,0\rangle \times \langle 1,2,2\rangle = \langle 2,-2,1\rangle$$

The plane goes through the origin, so is is all vectors perpendicular to this $\vec{n}$:

$$\boxed{\{\text{all } \vec{r} \text{ s.t. } \vec{r} \cdot \langle 2, -2, 1\rangle = 0\}}$$

c. *We already have that $\langle 1, 1, 0\rangle$ is perpendicular to $\vec{n}$. The rotation by $90°$ must be perpendicular to both, so it must be parallel to $\langle 1, 1, 0\rangle \times \vec{n} = \langle 1, -1, -4\rangle$. But this vector is too long by a factor of $\|\vec{n}\|$: the length of $\vec{a} \times \vec{b}$ is $\|\vec{a}\| \cdot \|\vec{b}\| \cdot \sin(\theta)$ (and for us $\theta = \pi/2$ because the vectors are already perpendicular), whereas the length of the image vector should just be the same as the length of the original vector. Dividing by $\|\vec{n}\| = 3$ gives*

$$\boxed{\text{image} = \langle \frac{1}{3}, -\frac{1}{3}, -\frac{4}{3}\rangle}$$