Math 53 Quiz 8

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Name: ____

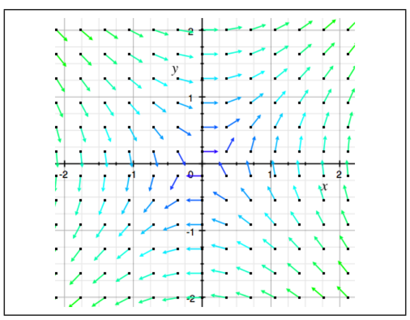
Time (circle one): 12:10 - 1:00 3:10 - 4:00

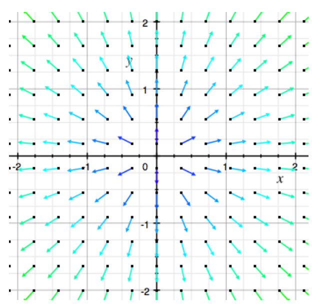
Please use extra paper as necessary. For each part, partial credit will be assigned based on correct work (you do need to show some work, enough so that I know how you solved the problem). Please simplify and $\boxed{\textit{box}}$ *your answers.*

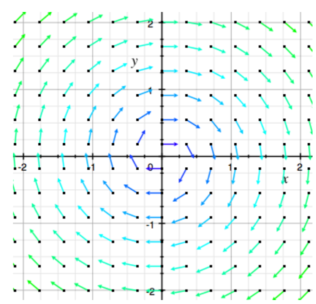
Consider the vector field:

$$\vec{v}(x, y) = y\vec{\imath} + x\vec{\jmath}$$

a. (1 pt) Which of the following is a graph of $\vec{v}$?







(Note: My graphing calculator displays only the direction of vectors, normalizing each to a unit vector, and displaying the magnitude as a color. But this is a black-and-white printer.)

b. (5 pt) Consider the path $\vec{r}(t) = (x(t), y(t)) = (1 + t, 3 - t)$ as t ranges from 0 to 1. Compute directly the line integral

$$\int_{\text{path}} \vec{v}(\vec{r}) \cdot d\vec{r}$$

(I.e. set up the integral in terms of t and use the methods of 13.2, but not 13.3.)

We plug in $x(t)$ and $y(t)$ into $\vec{v}(\vec{r})$, and $d\vec{r} = \frac{d\vec{r}}{dt}dt = \langle 1, -1\rangle dt$. Then the integral is

$$\int_{t=0}^{t} [(3-t)\vec{\imath} + (1+t)\vec{\jmath}] \cdot [\vec{\imath} - \vec{\jmath}]\, dt = \int_{t=0}^{1} (3 - t - 1 - t)\, dt = 2 - 1 = \boxed{1}$$

c. (4 pt) Is there a function $F(x, y)$ so that $\vec{v}(x, y) = \vec{\nabla} F$? If so, find such a function, and evaluate $F(2, 2) - F(1, 3)$. If not, explain why not.

If $\vec{v} = P\vec{\imath} + Q\vec{\jmath}$*, then we compare* $\partial P/\partial y \stackrel{?}{=} \partial Q/\partial x$*, and sure enough these match. Indeed, we have* $\partial F/\partial x = y$*, so* $F(x, y) = xy + c(y)$*, where* $c(y)$ *is an unknown "constant", i.e. it is a constant with respect to* x*. Alternately,* $\partial F/\partial y = x$*, so* $F(x, y) = xy + c'(x)$*, where* c' *is another unknown "constant". So* $\boxed{F(x, y) = xy}$ *works. Then* $\boxed{F(2, 2) - F(1, 3) = 4 - 3 = 1}$*, which is exactly the integral of* $\vec{v}$ *for any path starting at* $(1, 3)$ *and ending at* $(2, 2)$*, by the Fundamental Theorem of Calculus.*