MATH 53 10 March 08
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(1) Calculate the following double integrals:

(a) $\int_{y=1}^{2}\int_{x=0}^{2}(y+2xe^{xy})\,dx\,dy$

(b) $\int_0^1\int_0^1 ye^{xy}\,dx\,dy$

(c) $\iint_R \frac{1}{(x-y)^2}\,dA$ where $R=\{(x,y): 0\le x\le 1,\ 2\le y\le 4\}$

(2) Find the volume under the paraboloid $z=x^2+4y^2$ and above the rectangle $R=[0,2]\times[1,4]$ in the xy-plane

(3) The double integral $\int_0^1\int_0^1 \frac{1}{1-xy}\,dx\,dy$ is improper (why?). Expand it as an infinite series (Hint: $\frac{1}{1-r}=?$) and integrate term-by-term to express the value as an infinite series. Does this series converge?

(4)(a) For any real numbers (a_1,a_2), show that

$$\frac{1}{4}\left(|a_1+a_1|+|a_1+a_2|+|a_2+a_1|+|a_2+a_2|\right)\ge\frac{1}{2}\left(|a_1|+|a_2|\right)$$

(b) Given that for any n-tuple $(a_1,a_2,\dots,a_n)$ we have

$$\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}|a_i+a_j|\ge\frac{1}{n}\sum_{i=1}^{n}|a_i|,$$

show that $\int_0^1\int_0^1|f(x)+f(y)|\,dx\,dy\ge\int_0^1|f(x)|\,dx$ for any continuous function $f(x)$ on $[0,1]$.