MATH 53   13 Feb 08
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(1) Recall that the <u>curvature</u> of a curve $\vec{r}(t)$ is defined as

$$\kappa = \left|\frac{d\vec{T}}{ds}\right|$$

where $\vec{T} = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|}$ is the unit tangent vector.

(a) What is the curvature of the circle

$$x = \frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{6}}\sin\theta$$

$$y = -\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{6}}\sin\theta$$

$$z = -\frac{2}{\sqrt{6}}\sin\theta \qquad ?$$

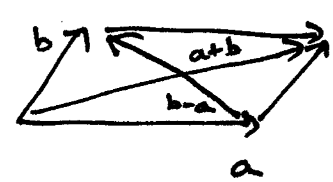
(b) What about the helix

$$x = \cos\theta + \frac{1}{\sqrt{3}}\sin\theta + 2\theta$$
$$y = -\cos\theta + \sin\theta\cdot\frac{1}{\sqrt{3}} + 2\theta$$
$$z = -\frac{2}{\sqrt{3}}\sin\theta + 2\cdot\theta \qquad ?$$

Remark: there is also a <u>vector curvature</u> $\vec{K} = \frac{d\vec{T}}{ds}$.

(2) Prove the following facts from geometry:

(a) A parallelogram is a <u>rectangle</u> (i.e. $\vec{a}\perp\vec{b}$) if and only if the diagonals have the same length



(b) A parallelogram is a <u>rhombus</u> (i.e. $\|a\| = \|b\|$) if and only if the diagonals are perpendicular

③ Cross product

A product $*$ is called associative if, for any inputs a, b, and c, we have $(a*b)*c = a*(b*c)$.

For example, usual multiplication, matrix multiplication, and function composition are associative.

(a) Find an example that shows that $\times$ (the vector cross-product) is not associative.

Thus, $\vec{v}_1 \times \vec{v}_2 \times \ldots \times \vec{v}_n$ is not defined for $n \geq 3$, unless we pick an association, e.g.

$$\left(\vec{v}_1 \times (\vec{v}_2 \times \vec{v}_3)\right) \times \vec{v}_4 \quad \text{or} \quad (\vec{v}_1 \times \vec{v}_2) \times (\vec{v}_3 \times \vec{v}_4)$$

When are two associations equal? Certainly, if

$$\vec{v}_1 = \vec{v}_2 = \ldots = \vec{v}_n$$

then both are 0.

(b) Prove that, ~~the~~ given two associations of ~~[illegible]~~ $\vec{v}_1 \times \vec{v}_2 \times \ldots \times \vec{v}_n$, then there is an assignment of $\hat{\imath}$, $\hat{\jmath}$, or $\hat{k}$ to each of the vectors $\vec{v}_i$ so that the two associations are equal and non-zero.