

MATH 53 22 Feb 08
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① (a) What are the level curves of the function
 $f(x, y) = \langle x, y \rangle \cdot \langle a, b \rangle$
for some vector $\langle a, b \rangle$?

(b) What are the level surfaces of the function
 $f(x, y, z) = \langle x, y, z \rangle \cdot \langle a, b, c \rangle$?

② (a) Sketch the function (i.e. sketch its graph)

$$f(x, y) = x^y$$

What is the domain of this function?

(b) Limits in multiple variables are rather hairy. We should define 0^0 as $\lim_{x, y \rightarrow 0} x^y$. Show

that this definition fails, by finding two paths to 0 in the domain of f so that the limits along those paths are different.

(c) Make sense of the statement " $0^0 = 1$ if they are the same 0".

③ What about

$$\lim_{x, y \rightarrow 0} \frac{xy^2}{x^2 + y^4} ?$$

Evaluate this limit along any path $y = mx, x \rightarrow 0$.

Evaluate it along $x = y^2, y \rightarrow 0$.