MATH 53 27 Feb 08

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① Consider the function on three variables

$$f(x,y,z) = x^2 + y^2 + xy + yz + xz$$

(a) If we restrict our attention to the level surface

$$f(x,y,z) = 0$$

we can find a function $g(x,y)$ so that this surface is the graph of

$$z = g(x,y).$$

Do this.

(b) Slice this surface along the plane $y=1$; we get a space curve

$$(x,\ 1,\ g(x,1))$$

parameterized by x. Find the tangent vector $\vec{v}$ to this curve at $x=1$. <u>Hint:</u> $\frac{d}{dx}(g(x,1)) = \frac{\partial g}{\partial x}$ at $y=1$.

(c) Slice the surface along $x=1$; ~~they~~ find the tangent vector $\vec{w}$ to $(1, y, g(1,y))$ at $y=1$.

(d) What is $\vec{v} \times \vec{w}$?

(e) What is the vector of partial derivatives

$$\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

at $(x,y,z) = (1, 1, g(1,1))$? How does this vector relate to your answer to part (d)?

(f) Interpret this all geometrically.

② Generalize the relationship you found in ①ⓔ to generic function $f(x, y, z)$ and replacing $(x, y) = (1,1)$ by a generic point.

<u>Hint:</u> you do not need to solve for z as a function of x and y: use implicit differentiation to find $\frac{\partial g}{\partial x}$ and $\frac{\partial g}{\partial y}$ in terms of $f, x, y, z, \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$, and $\frac{\partial f}{\partial z}$.

(You know that, on a level surface, $f(x, y, g(x,y))$ is a constant.)