MATH 53 APRIL 30, 2008
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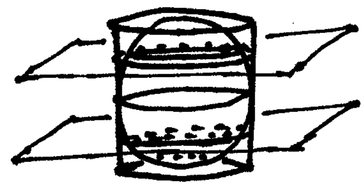
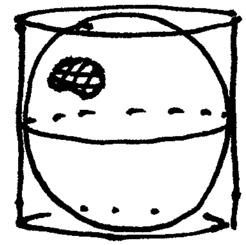
① Warm-up: Consider a shere inscribed in a cylinder, and slice the figure by two planes perpendicular to the axis ~~of~~ the cylinder. Show that the surface area of the part of the sphere between the two planes is equal to the surface area of the part of the cylinder.

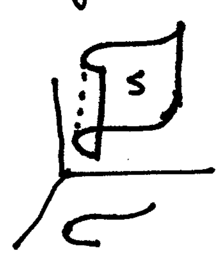




Find a similer statement for an arbitary region on the sphere and corresponding region on the cylinder (i.e. what is the correct corresponding region?

② (a) In the special case when your surface S is verticle, with constant height and your vector field does not depend on z, relate the surface integral of a vector field over S to a line integral.

(b) When S is a cylinder over an arbitrary simple closed curve, and $\vec{F}$ is a vector field that does not depend on z, prove the divergence theorem that



$$\oiint_S \vec{F} \cdot d\vec{S} = \iiint_{\text{interior of } S} (\operatorname{div} \vec{F})\, dx\,dy\,dz$$

Hint: the top and bottom cancel.

(c) In the case in (a), prove Stoke's theorem:

$$\iint_S (\operatorname{curl} \vec{F}) \cdot d\vec{S} = \oint_{\partial S} \vec{F} \cdot d\vec{r}$$