MATH 53 30 Jan 08
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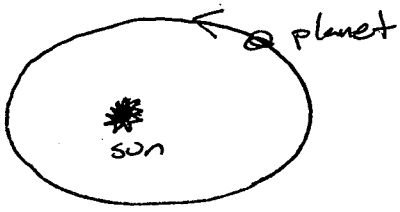
(1) Kepler, based on detailed measurements, observed that planets orbit in ellipses with the sun at a focus:



$$r = \frac{e}{1 + e\cos\theta}, \quad e < 1$$

Kepler also observed that the area

$$A(\alpha, \beta) = \frac{1}{2}\int_\alpha^\beta r^2 \, d\theta$$

traced out as the planet moves from angle α to angle β depends only on the amount of time passed.

(a) Find $A(\alpha, \beta)$, and find a parameterization $\theta(t)$ of the ellipse so that $A(\theta(t), \theta(t+s))$ depends only on the time difference s.

(b) Since gravity pulls only towards the sun, the angular momentum

$$l = r^2 \frac{\partial\theta}{\partial t}$$

must be constant. Check that $\frac{\partial l}{\partial t} = 0$ for your parameterization.

In fact, $\frac{\partial l}{\partial t} = 0$ if and only if the areas are constant.