MATH 53   6 Feb 08

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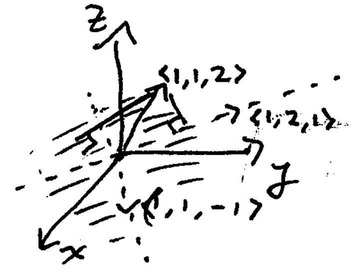
① Consider the dot product $\langle a,b,c\rangle \cdot \langle x,y,z\rangle = ax+by+cz$. Later in the semester, we will define the partial derivatives $\frac{\partial}{\partial x}$, $\frac{\partial}{\partial y}$, and $\frac{\partial}{\partial z}$, which each hold the other two variables constant, and differentiate. (For exaple, $\frac{\partial}{\partial x}(x+xy) = 1+y$, $\frac{\partial}{\partial y}(x+xy) = x$.) We can make a "vector" $\partial = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\rangle$.

(a) Calculate $\frac{\partial}{\partial x}$, $\frac{\partial}{\partial y}$, and $\frac{\partial}{\partial z}$ of $\langle a,b,c\rangle \cdot \langle x,y,z\rangle$, and put these into a vector. What vector is this?

(b) What about for $\langle x,y,z\rangle^2 = \langle x,y,z\rangle \cdot \langle x,y,z\rangle$?

This is why the dot product deserves to be called a product.

② (a) What is the orthogonal projection of the vector $\langle 1,1,2\rangle$ onto the line spanned by $\langle 1,2,1\rangle$?



(b) How would you project the vector $\langle 1,1,2\rangle$ onto the plane generated by $\langle 1,2,1\rangle$ and $\langle 1,1,-1\rangle$?

③ Prove Cauchy-Schwartz inequality, that

$$\vec{v} \cdot \vec{w} \leq \|\vec{v}\| \cdot \|\vec{w}\|$$

where $\|\vec{v}\| = \sqrt{\vec{v}\cdot\vec{v}}$.