

Rehearsal midterm 2

1. z is a function defined implicitly

$$\text{by } x^2 + xz + y^2 + yz + z^2 = 5$$

what are $z_x(1, 1)$, $z_y(1, 1)$?

$$\text{Sol'n } \partial_x) \quad 2x + xz_x + z + yz_x + 2zz_x = 0$$

$$2 + z_x + 1 + 1z_x + 2z_x = 0$$

$$3 + 4z_x = 0 \quad z_x = -\frac{3}{4}$$

$$\partial_y) \quad 2y + yz_y + z + 2zz_y = 0$$

$$2 + z_y + 1 + 2z_y = 0$$

$$z_y = -1$$

2. The elevation of terrain is given by $z = x^2 - y^2$ and the "heart" of a mountain lion hunting "K.R." is given by parametric eqns

$$x(t) = \frac{1}{2} + \cos t, \quad y(t) = \frac{1}{2} + \sin t.$$

The lion is going to hog K.R. at the lowest point on his heart. Where is that?

3. a) Compute ∇f for $f(x, y, z) =$

$$x^2 + xz + y^2 + yz + z^2$$

b) What is the equation of the tangent

plane to the surface $f(x, y, z) = 5$

at $(1, 1, 1)$?

c) Compute the "linearized approximation"

to $f(1.01, 1.005, 1.02)$.

4. Compute $\iint_D \sqrt{x^2+y^2} \, dA,$

where D in terms of polar coord is

$$0 \leq r \leq 2 + \sin 4\theta.$$

5. Compute $\iiint_E \frac{r^2}{x^2+y^2+z^2} \, dV$

where E is the shell between

spheres of radii $\frac{1}{2}$ and 1 centered

about $(0, 0, 0)$.

6. In a movie theatre each person has to wait for some time t in the "candy" line and some time s in the "poor" line.

The distribution of t is $f(t) = e^{-\frac{t}{10}}$ for $t > 0$ and distribution of s is $g(s) = \frac{1}{30} e^{-\frac{s}{30}}$

for $s > 0$. What fraction of people must endure a total wait $t + s > 60$?