Math 53 Quiz 10

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Name: ____

Time (circle one): 12:10 - 1:00 3:10 - 4:00

Please use extra paper as necessary. For each part, partial credit will be assigned based on correct work (you do need to show some work, enough so that I know how you solved the problem). Please simplify and $\boxed{\textit{box}}$ *your answers.*

a. (3 pts) Given a continuously twice-differentiable (scalar) function f, prove the following product rule:

$$\vec{\nabla} \cdot \left(f\, \vec{\nabla} f \right) = \left(\vec{\nabla} f \right)^2 + f\, \nabla^2 f$$

b. (3 pts) Hence, for a region R (in the plane) with boundary ∂R (and where $\vec{n}$ is a unit vector perpendicular to the boundary, ds is an infinitesimal piece of boundary, dA is the area form on R, etc.), on which f is defined and continuously twice-differentiable, prove:

$$\oint_{\partial R} f\, \vec{\nabla} f\, \vec{n} ds = \iint_R (\vec{\nabla} f)^2\, dA + \iint_R f\, \nabla^2 f\, dA$$

c. (2 pts) Now assume that f is a harmonic function, i.e. $\nabla^2 f = 0$, and that f is 0 on the boundary ∂R. Then what can you conclude about the value of $\vec{\nabla} f$ inside the region R? (Hint: The integral of a positive function is positive.)

d. (2 pts) Hence, what are all possible harmonic functions f on a region that vanish on the boundary of that region? (Hint: What do you know about functions with zero derivative?)