Math 53 Quiz 2

8 February 2008

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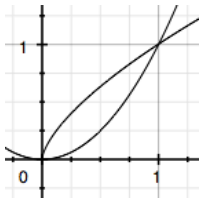
Name: _____

Time (circle one): 12:10 - 1:00 3:10 - 4:00

1. (4 pts) Consider the parameterized curves

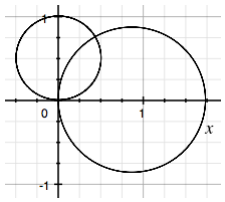
$$\left\{ \begin{array}{rcl} x_1 &=& t \\ y_1 &=& t^2 \end{array} \right\} \text{ and } \left\{ \begin{array}{rcl} x_2 &=& t^3 \\ y_2 &=& t^2 \end{array} \right\}$$



Calculate the velocity vectors $\vec{v}_1 = \langle \frac{dx_1}{dt}, \frac{dy_1}{dt} \rangle$ and $\vec{v}_2 = \langle \frac{dx_2}{dt}, \frac{dy_2}{dt} \rangle$ at the intersection point $t = 1$, and calculate their dot product.

2. (6 pts) We can do the same thing in polar coordinates. Consider two curves

$$\left\{ \begin{array}{rcl} r_1 &=& \sin t \\ \theta_1 &=& t \end{array} \right\} \text{ and } \left\{ \begin{array}{rcl} r_2 &=& \sqrt{3} \cos t \\ \theta_2 &=& t \end{array} \right\}$$



In polar coordinates, we can still calculate velocities. Calculate the velocity vectors $\vec{v}_1 = \langle \frac{dr_1}{dt}, \frac{d\theta_1}{dt} \rangle$ and $\vec{v}_2 = \langle \frac{dr_2}{dt}, \frac{d\theta_2}{dt} \rangle$ at the point of intersection $t = \pi/3$.

However, the dot product between velocity vectors in polar coordinates is more complicated, because it depends on the value of $r(t)$ at the point of intersection:

$$\langle \frac{dr_1}{dt}, \frac{d\theta_1}{dt} \rangle \cdot \langle \frac{dr_2}{dt}, \frac{d\theta_2}{dt} \rangle = \frac{dr_1}{dt}\frac{dr_2}{dt} + (r(t))^2 \frac{d\theta_1}{dt}\frac{d\theta_2}{dt}$$

Calculate this dot-product for these two curves.

3. (1 pt bonus) Interpret the value of the dot-product in the previous question geometrically: what is it telling you about the two vectors?

Please use extra paper as necessary. For each part, partial credit will be assigned based on correct work (you do need to show some work, enough so that I know how you solved the problem). Please simplify and $\boxed{\text{box}}$ your answers.