Math 1B Final Exam
Friday, 15 August 2008

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Name: ______________________________

Problem Number	1	2	3	4	5	6	7	Total
Score								
Maximum	10	20	10	15	15	15	15	100

Please do not begin this test until 8:10 a.m. The test ends exactly at 10 a.m.
As always, show work for partial credit. Please box your final answers.

1. (10 pts – 5 questions, 2 pts each) Determine whether the following statements are true or false. Full points will be awarded for the correct answer; partial credit may be awarded for useful thoughts without the correct answer. Throughout, a_n, b_n, and c_n are unknown sequences of (possibly negative) real numbers.

   (a) TRUE or FALSE: If $\displaystyle\lim_{n\to\infty}\left|\frac{a_n}{b_n}\right| = 0$ and $\displaystyle\sum_{n=1}^{\infty} b_n$ converges absolutely, then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges absolutely.

   (b) TRUE or FALSE: If $\displaystyle\sum_{n=1}^{\infty} a_n$ converges absolutely and $\displaystyle\sum_{n=1}^{\infty} b_n$ converges conditionally, then $\displaystyle\sum_{n=1}^{\infty}(a_n + b_n)$ converges conditionally.

   (c) TRUE or FALSE: If $\displaystyle\sum_{n=1}^{\infty} c_n(-4)^n$ converges, then $\displaystyle\sum_{n=1}^{\infty} c_n 3^n$ converges absolutely.

   (d) TRUE or FALSE: If $a_n \le b_n$ for every n and $\displaystyle\sum_{n=1}^{\infty} b_n$ converges, then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.

   (e) TRUE or FALSE: If $\displaystyle\lim_{n\to\infty}[a_{n+1} - a_n] \ne 0$, then $\displaystyle\lim_{n\to\infty} a_n$ does not converge.

2. (20 pts – 4 questions, 5 pts each) Determine whether the following series converge absolutely, converge conditionally, or diverge. Explain how you know.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n + \frac{1}{n+1}}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n \arctan n}{3^n}$

(c) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{n(n+2)}{(n+3)^2}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \overbrace{2\cdot 5\cdot 8\cdot \ldots \cdot (3n-4)\cdot(3n-1)}^{n \text{ numbers}}}{\underbrace{2\cdot 6\cdot 12\cdot 20\cdot 30\cdot \ldots \cdot (n-1)n\cdot n(n+1)}_{n \text{ numbers}}}$

3. (10 pts) Find the radius and interval of convergence of the following power series.

$$\sum_{n=0}^{\infty} \frac{(n+1)}{4^n (n+2)^2} (x-2)^n$$

4. (a) (10 pts) Find a power-series representation centered at $x = 4$ for the function $\sqrt{x}$. You may use any method you wish: manipulating known power series, Taylor's theorem, etc.

(b) (5 pts) What is the radius of convergence of your answer to part (a)? (You do not need to decide if the series converges at the endpoints.)

5. (a) (5 pts) Find a power-series representation centered at $x = 0$ for the function $\sin(x/5)$. You may use any method you wish: manipulating known power series, Taylor's theorem, etc.

(b) (10 pts) For what n does the nth Taylor polynomial correctly estimate $\sin(2/5)$ to within an error of 0.001? Compute the first three digits of $\sin(2/5)$.

6. (15 pts) Solve the following initial value problem, by assuming that the solution can be represented by a power series.

$$xy'' + y' - xy = 0, \;\; y(0) = 1, \; y'(0) = 0$$

7. (a) (10 pts) Use the Trapezoid Rule with three subdivisions to estimate $\ln 4 = \int_1^4 \frac{dx}{x}$. What is the expected error of this estimate? Is the estimate too high or too low (hint: draw a picture)? Give a decimal range of possible values for $\ln 4$ based on your estimate.

(b) (5 pts) For what n does the Midpoint Rule with n subdivisions estimate $\int_1^4 \frac{dx}{x}$ to within an error of 0.01?

8. (0 pts) Thanks for the great summer! Use this page if you need the extra space.