

# Math 1B Handout: Final Exam Review

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You will have 110 minutes to complete the final exam. You may bring one page (front and back) of notes. You are not allowed a calculator.

## Outline of final

- (10 pts – 5 questions, 2 pts each) Determine whether the following statements are true or false. Full points will be awarded for the correct answer; partial credit may be awarded for useful thoughts without the correct answer.
- (20 pts – 4 questions, 5 pts each) Determine whether the following series converge absolutely, converge conditionally, or diverge. Explain how you know.
- (10 pts) Find the radius and interval of convergence of the following power series.
- (10 pts) Find a power-series representation centered at  $x = 4$  for the function  $f(x)$ . You may use any method you wish: manipulating known power series, Taylor's theorem, etc.
  - (5 pts) What is the radius of convergence of your answer to part (a)? (You do not need to decide if the series converges at the endpoints.)
- (5 pts) Find a power-series representation centered at  $x = 0$  for the function  $g(x)$ . You may use any method you wish: manipulating known power series, Taylor's theorem, etc.
  - (5 pts) For what  $n$  does the  $n$ th Taylor polynomial correctly estimate  $g(2)$  to within an error of 0.0005? Compute the first three digits of  $g(2)$ .
- (15 pts) Solve the following initial value problem, by assuming that the solution can be represented by a power series.
- (10 pts) Use the Trapezoid Rule with three subdivisions to estimate  $\int_a^b h(x)dx$ . What is the expected error of your estimation? Is the estimate too high or too low (hint: draw a picture)? Give a decimal range of possible values for  $\int_a^b h(x)dx$  based on your estimate.
  - (5 pts) For what  $n$  does the Midpoint Rule with  $n$  subdivisions estimate  $\int_a^b h(x)dx$  to within an error of 0.01?

## What you need to know

### Sequences Chapter 8 Review 1–8

- How to compute limits of sequences given as functions
- Monotone sequence theorem

### Series — general Chapter 8 Review 9–24

- Difference between a series and a sequence
- Divergence Test
- Alternating Series Test
- Absolute v.s. Conditional Convergence

### Comparison and Integral Tests Chapter 8 Review 9–24

- Limit-comparison Test
- $p$ -Test

### Power Series Chapter 8 Review 36–50, Chapter 17 Review 15–16

- Ratio Test
- Radius and Interval of Convergence
- Manipulations: Adding, Differentiating/Integrating, Re-indexing
- Differential Equations with Power Series
- Taylor's Theorem
- Known series: trigonometric (sin, cos, arctan), exponential (exp, ln), rational (geometric, binomial theorem)

### Estimations Chapter 8 Review 52–54, Chapter 6 Review 57–59

- Trapezoid and Midpoint Rules, and error formulas for them
- Remainder Theorem in Taylor's Theorem