# Math 1B Handout:
# Power Series Solutions to Differential Equations

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Tuesday, 5 August 2008

1. Rewrite (i.e. re-index) each of the following expressions to get each into the form $\sum d_n x^n$:

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+2}}{2^{n+1}} x^{n+3}$ (b) $\displaystyle\sum_{n=1}^{100} n x^{n-1}$ (c) $\displaystyle\sum_{n=2}^{\infty} \frac{n(n-1)}{2} x^{n-2}$

2. Suppose $y(x) = \sum_{n=0}^{\infty} c_n x^n$. Express $y'$, $y''$, $xy$, and $xy'$ as power series of the form $\sum d_n x^n$. (I.e. in each answer, the $n$th term in the sum should be a number (depending on $n$ and the $c_m$s) times $x^n$.)

3. Let $y = \sum_{n=0}^{\infty} c_n x^n$. In each of the following expressions, substitute this power series and re-write the results as a power series of the form $\sum d_n x^n$:

   (a) $y' - 6y$
   (b) $xy'' - y$

4. Unwind the following recurrence relations:

   (a) $(n+1)a_{n+1} = a_n$. Write $a_n$ in terms of $a_0$.
   (b) $c_{n+2} = -\frac{c_n}{(n+1)(n+2)}$. Write $c_n$ in terms of $c_0$ or $c_1$.
   (c) $n^2 a_n + a_{n-2} = 0$ and $a_1 = 0$. Write $a_n$ in terms of $a_0$.

5. Use power series to find the general solution to $y' - y = 0$. Is the answer what you expected?

6. Use power series to solve the differential equation:

   (a) $y' = xy$ (b) $(x-2)y' + 2y = 0$ (c) $y'' = y$

   Also solve each of the above equations without using power series. In this way, derive power series representations for all those functions. Find the interval of convergence of each of the above power series.