# Math 1B Optional "Term Paper"

GSI: Theo Johnson-Freyd
http://math.berkeley.edu/∼theojf/08Summer1B/

Due: Monday, 11 August 2008

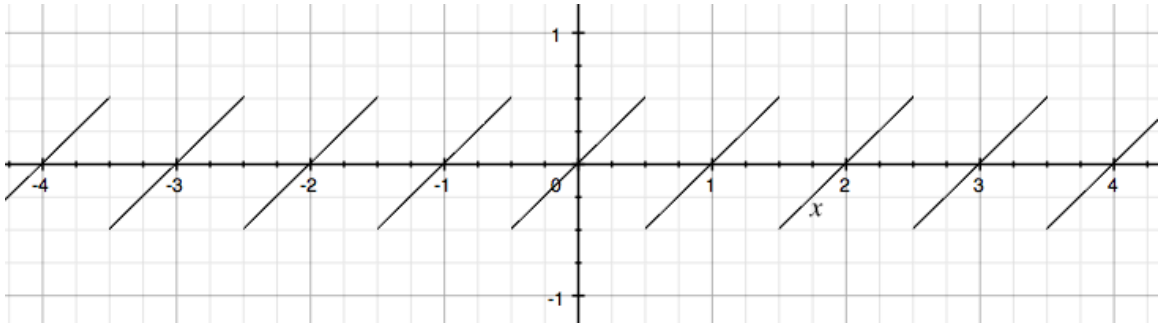
Students who performed sufficiently poorly on the midterm may decide to complete this assignment, which can replace part or all of their midterm score. The assignment is not extra credit — students who did well on the midterm will not have their grades raised by completing this assignment. I expect it to be quite a lot of work.

The assignment consists of writing a "term paper" for the course, and then being prepared for a half-hour "oral exam" on the term paper. I will not grade the paper for writing quality per se, but I expect the paper to be neat, correctly spelled and punctuated, and generally well written. A typed paper is strongly preferred, although you may decide to add the equations by hand. The paper must demonstrate that you do in fact know all the material you failed to show on the midterm. I recommend that you work with others on the paper, and use any resources at your disposal — internet, tutors, books — but the paper writing must be your own, and you must know the material well enough to present it in an oral exam setting.

The paper should be an exploration of a particularly simple model of an AM radio. In particular, the "radio" you should consider is a simple LCR series circuit, in which each component (the inductance, the capacitance, and the resistance) may be adjusted. The antenna plays the roll of the power source. This radio has no transistors, no amplifier, and no speaker.

So, assume you have an LCR circuit which is initially in its rest state: there is no current, and no charge on the capacitor. Now assume that a signal is turned on. Explain how the circuit responds to a pure tone $E(t) = \sin \omega t$, for various values of $\omega$. (Remember that the values of $L$, $C$, and $R$ are also all variable. You should explain all possible responses, and include the various values for which they hold. Consider special cases, like when $R = 0$ or $R^2 = 4L/C$, and when $\omega^2 = 1/LC$. There may be other special cases as well.)

Also explain how the circuit responds to a "violin" sound (this is close to what a pure note on a violin looks like when the sound wave is graphed; it's also often described as a "sawtooth" sound, because the graph looks like the serrated edge of a saw):




You should assume that the circuit responds in such a way so that the current and the charge on the capacitor are both continuous functions of time. Of course, you should explore how the response depends on the values of $L$, $C$, and $R$.

What if the pure-tone and violin tones are suppressed by an exponential $e^{\alpha t}$? Explain how to modify a circuit so that it responds to a suppressed signal as if the original circuit were to receive an unsuppressed signal.

Pick a few other reasonable signals. Variations of $E(t) = 1/(1+\cos^2 t)$ are good. I will definitely ask you to solve an initial-value problem with an LCR circuit and a signal like $1/(1+\cos^2 t)$ in the oral-exam component, so be sure you know how.

Lastly, it would be awesome to include some discussion of how reasonable this model of a radio is (I expect the answer is "not very"). For example, an actual crystal radio is a parallel circuit rather than a series circuit. At the very least, look up enough electrical engineering to write down the differential equation explaining how a parallel LC circuit with some resistance (but no demodulator or headphones) responds to a power source. If this differential equation is of any of the types of differential equations we discussed in class, solve it for a few possible incoming signals, and at least for the pure tone.

In general, I should emphasize that this is an open-ended essay topic. It will require a lot of work, and also it will require learning things from outside the scope of the course. If you find yourself getting behind in the homework to work on the essay, then probably you should not write the essay. If the essay as outlined above is too hard or long, it's fine to just answer some of the questions, but be sure that you convince me you know the calculus. I will not grade these essays from a rubric or out of a particular score, except to see if you have learned all the calculus from the first half of the course.