Math 1B Quiz 12

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Name: ____ Score: /10

You have twenty minutes (plus the break) to complete the following closed-note open-chalkboard quiz. Partial credit will be awarded for correct work, and no points will be given for simply writing down the correct answer. Please box your final answers.

1. (0 pts) Favorite math you learned all summer?

   The generalized product and chain rules.

2. (10 pts) Find a Taylor Series expansion of the following function centered at $a = 9$:

$$f(x) = \frac{1}{\sqrt{x}}$$

Find the interval of convergence of your series. Convergence is always absolute inside the radius of convergence; for each boundary in your series, state whether the series converges absolutely, converges conditionally, or diverges.

We calculate derivatives:

n	$f^{(n)}(x)$	$c_n(x-9)^n = f^{(n)}(9)\,(x-9)^n/n!$
0	$x^{-1/2}$	$1/3$
1	$-\frac{1}{2}x^{-3/2}$	$-(1/2)(1/27)\,(x-9)$
2	$\frac{3}{4}x^{-5/2}$	$(3/4)(1/3^5)\,(x-9)^2/2$
3	$-\frac{15}{8}x^{-7/2}$	$-(1\cdot 3\cdot 5/2^3)(1/3^7)\,(x-9)^3/3!$
$\vdots$		$\vdots$
n	$(-1)^n\frac{1\cdot 3\cdot \ldots \cdot (2n-1)}{2^n}x^{-(2n+1)/2}$	$(-1)^n\dfrac{1\cdot 3\cdot \ldots \cdot (2n-1)}{2^n 3^{2n+1} n!}(x-9)^n$

Thus, the sum is

$$\frac{1}{\sqrt{x}} = \sum_{n=0}^{\infty}(-1)^n\frac{1\cdot 3\cdot \ldots \cdot (2n-1)}{2^n 3^{2n+1} n!}(x-9)^n = \sum_{n=0}^{\infty}(-1)^n\frac{(2n)!}{2^{2n}3^{2n+1}(n!)^2}(x-9)^n$$

The radius of convergence is

$$R = \lim_{n\to\infty}\left|\frac{c_n}{c_{n+1}}\right| = \lim_{n\to\infty}\frac{2^2 3^2 (n+1)^2}{(2n+2)(2n+1)} = 9$$

The interval is centered at 9, *so the endpoints are* 0 *and* 18. *By Stirling's formula, the series diverges at* $x = 0$ *with* $\sum 1/\sqrt{n}$, *and converges conditionally at* $x = 9$ *with* $\sum(-1)^n/\sqrt{n}$.