Math 1B Quiz 10

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Name: ____ Score: /10

You have twenty minutes (plus the break) to complete the following closed-note open-chalkboard quiz. Partial credit will be awarded for correct work, and no points will be given for simply writing down the correct answer. Please box your final answers.

1. (5 pts — 1 pt each) Determine whether each of the following statements is true or false.

   (a) If $\sum_{n=0}^{\infty} c_n 8^n$ converges, then so does $\sum_{n=0}^{\infty} c_n(-6)^n$.
   True. Because $R \geq 8$, and hence $|-6| < R$.

   (b) If $\sum_{n=0}^{\infty} c_n 8^n$ converges, then so does $\sum_{n=0}^{\infty} c_n(-8)^n$.
   False. E.g. $c_n = (-1)^n/n8^n$.

   (c) If $\sum_{n=0}^{\infty} c_n x^n$ has radius of convergence equal to R, then $\sum_{n=0}^{\infty} c_n R^n$ converges conditionally.
   False. E.g. $c_n = 1/n^2$ or $c_n = 1$.

   (d) The radius of convergence of a power series $\sum_{n=0}^{\infty} c_n x^n$ always equals the limit of c_{n+1}/c_n as n tends to ∞.
   False. Numerous errors: the fraction is reversed, there are no absolute values, and the limit may not exist.

   (e) The radius of convergence of $\sum_{n=0}^{\infty} c_n x^n$ is twice the radius of convergence of $\sum_{n=0}^{\infty} c_n 2^n x^n$.
   True. If $|x| < R$ makes the series $\sum c_n 2^n x^n$ converge, then $|2x| < 2R$ makes $\sum c_n (2x)^n$ converge, hence $|x| < R$ makes $\sum c_n x^n$ converge.

2. (0 pts) What was the first thing you said this morning?

   "Grr, I don't want to get up."

3. (5 pts) Find the radius and interval of convergence of the following power series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n\, x^n}{n^2\, 5^n}$$

Radius of convergence:

$$\begin{aligned} R &= \lim_{n\to\infty} \left| \frac{c_n}{c_{n+1}} \right| \\ &= \lim_{n\to\infty} \left| \frac{\frac{1}{n^2 5^n}}{\frac{1}{(n+1)^2 5^{n+1}}} \right| \\ &= \lim_{n\to\infty} \frac{5^{n+1}\,(n+1)^2}{5^n\, n^2} \\ &= \lim_{n\to\infty} 5 \left(\frac{n+1}{n} \right)^2 \\ &= 5 \end{aligned}$$

At the boundaries $x = \pm 5$, the series is $\sum \pm 1/n^2$, which converges absolutely by the p-test with $p = 2$. Hence the interval of convergence is $\boxed{x \in [-5, 5]}$.