Math 1B Quiz 8

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Name: ____________________________________ Score: /10

You have twenty minutes (plus the break) to complete the following closed-note open-chalkboard quiz. Partial credit will be awarded for correct work, and no points will be given for simply writing down the correct answer. Please box your final answers. Use the back of the page as necessary.

1. (5 pts) Determine if the following sequence converges or diverges. If it converges, find the limit. If it diverges, explain how you know.

$$\left\{ \left(1 + \frac{1}{n}\right)^{2n} + \sqrt{\frac{1 + e^{-n}\cos n}{4 - \frac{1}{\ln n}}} \right\}_{n=2}^{\infty}$$

We assume the sequence converges, and try to compute the limit. If it diverges, we will presumably fail.

$$\begin{aligned}
\lim_{n\to\infty} \left[\left(1 + \frac{1}{n}\right)^{2n} + \sqrt{\frac{1 + e^{-n}\cos n}{4 - \frac{1}{\ln n}}} \right] &= \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^{2n} + \lim_{n\to\infty} \sqrt{\frac{1 + e^{-n}\cos n}{4 - \frac{1}{\ln n}}} \\
&= \left(\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^{n} \right)^2 + \sqrt{\lim_{n\to\infty} \frac{1 + e^{-n}\cos n}{4 - \frac{1}{\ln n}}} \\
&= e^2 + \sqrt{\frac{1 + \lim_{n\to\infty} e^{-n}\cos n}{4 - \lim_{n\to\infty} \frac{1}{\ln n}}} \\
&= e^2 + \sqrt{\frac{1+0}{4-0}} \\
&= \boxed{e^2 + \frac{1}{4}\text{: \textit{the sequence converges}}}
\end{aligned}$$

2. (0 pts) Can you make the currently schedule office hours times? What's a time during the week for office hours that you would actually attend?

   Yes, and I'm there every day.

3. (5 pts) Determine whether the following series converges or diverges. Explain how you know.

$$\sum_{n=1}^{\infty} \frac{n!\, e^n}{n^{n+1}}$$

Remembering Stirling's formula, we use the limit comparison test, comparing $(n!e^n)/n^{n+1}$ *with* $1/\sqrt{n}$*:*

$$\lim_{n\to\infty} \frac{(n!e^n)/n^{n+1}}{1/\sqrt{n}} = \lim_{n\to\infty} \frac{n!\, e^n}{n^n\, \sqrt{n}} = \sqrt{2\pi}$$

which is finite and positive. Thus $\sum \frac{n!\, e^n}{n^{n+1}}$ *converges if and only if* $\sum 1/\sqrt{n}$ *does. But* $\sum 1/\sqrt{n}$ *diverges by the p-test with* $p = 1/2$*, i.e. by the integral test comparing with* $\int_1^\infty dx/\sqrt{x}$*.*